March 26, 2003 Physics 202

EXAM 2

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

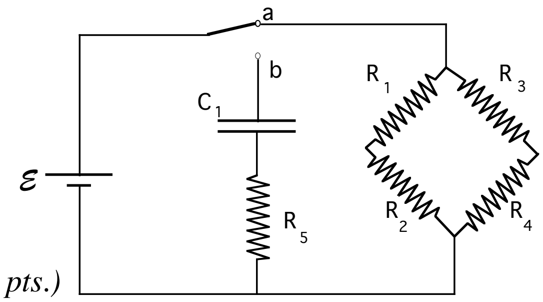
TOTAL: __________

Solution Key

Possibly useful information:

Permeability of Free Space:	μ_0	$= 4\pi \times 10^{-7}$ T m/A
Permittivity of Free Space:	ϵ_0	$= 8.854 \times 10^{-12}$ C^2/Nm^2
Electron charge:	e	$= -1.602 \times 10^{-19}$ C
Electron mass:	m_e	$= 9.109 \times 10^{-31}$ kg

PROBLEM 1



Resistors R_1=31 Ω, R_2=41 Ω, R_3=59 Ω, R_4=26 Ω, R_5=53 kΩ, and capacitor C_1=5.9 x 10^{-8} F are connected to a battery with emf $\mathcal{E}$ = 10.0 V as shown in the diagram at right. Initially the switch is in position a (as shown).

a. What is the total current produced by the battery? *(5 pts.)*

$R_1 + R_2 = 72\ \Omega$, $\qquad R_3 + R_4 = 85\ \Omega$

$$R_{eq} = \frac{1}{\frac{1}{72\,\Omega} + \frac{1}{85\,\Omega}} = 39\,\Omega \qquad I = \mathcal{E}/R_{eq} = 10.0\text{ V}/39\ \Omega = 0.26\text{ A}$$

0.26 A

b. What is the voltage across R_1? *(5 pts.)*

$$V_{R1} = \mathcal{E}\frac{R_1}{R_1 + R_2} = 10.0\text{ V}\ \frac{31\,\Omega}{72\,\Omega} = 4.3\text{ V}$$

4.3 V

c. Resistor R_3 is a uniform cylinder of length 2.5 cm and radius 0.83 cm. What is the resistivity of the material of which it is made? *(5 pts)*

$$\rho = \frac{RA}{\ell} = \frac{(59\,\Omega)(\pi)(0.0083\text{m})^2}{0.025\text{ m}} = 0.51\,\Omega\text{ m}$$

0.51 Ω m

d. At time t=0 the switch is moved to position b. How much time does it take for the voltage across R_5 to reduce to 37% of its value at t=0? *(5 pts)*

$\tau = R_5C_1 = (5.3 \text{ x } 10^4\ \Omega)(5.9 \text{ x } 10^{-8}\text{ F}) = 3.1$ ms

3.1 ms

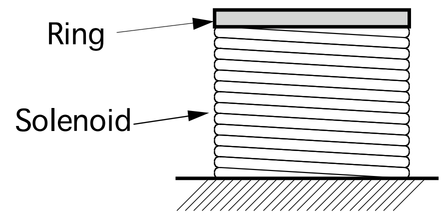
e. How much time elapses until the current is 1.0% of its value at t=0? *(5 pts.)*

$e^{-t/\tau} = 0.01 \qquad -t/\tau = \ln(0.01) = -4.61 \qquad t = 4.61\,(3.1\text{ ms}) = 14\text{ ms}$

14 ms

PROBLEM 2 Jumping Ring (almost)

A solenoid of radius r=8.6 cm and length l=18 cm has 150 turns of wire. It is placed on its end so that its axis is vertical. On top of the solenoid sits an aluminum ring of the same radius as the solenoid. The current through the solenoid is initially zero. At time t=0 the current begins to increase linearly with time, reaching 180 Amps after 0.10 s has elapsed. After this the current remains constant. (You may assume that all flux passing through the solenoid passes through the ring).



a. At t=0.10 s, what is the magnitude of the magnetic field inside the solenoid? *(5 pts.)*

$B = \mu_0 N I / \ell = (4\pi \times 10^{-7}\ \mathrm{Tm/A})(150)(180\ \mathrm{A})/(0.18\ \mathrm{m}) = 0.19\ \mathrm{T}$

0.19 T

b. At t=0.10 s, what is the magnetic flux passing through the surface of an imaginary cylinder, of radius 10 cm and length 20 cm, enclosing the solenoid, with an axis aligned with the solenoid axis? *(5 pts.)*

Gauss's Law for magnetism—net flux through any closed surface is zero.

0 Wb

c. At t=0.10 s, what is the magnetic flux passing through the aluminum ring? (You may assume the aluminum ring has not moved a significant distance from its original position.) *(5 pts.)*

$\Phi = BA = (0.19\ \mathrm{T})(\pi)(0.086\ \mathrm{m})^2 = 4.4 \times 10^{-3}\ \mathrm{Wb}$

4.4×10^{-3} Wb

d. At t=0.10 s, what is the EMF in the ring? *(5 pts.)*

$\mathcal{E} = -d\Phi/dt = -\Delta\Phi/\Delta t = (4.4 \times 10^{-3}\ \mathrm{Wb})/(0.1\ \mathrm{s}) = 4.4 \times 10^{-2}\ \mathrm{V}$

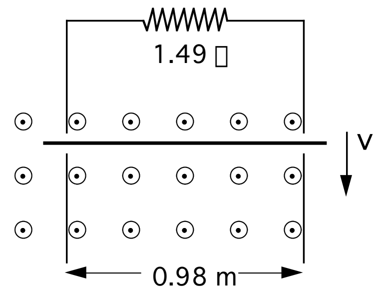
4.4×10^{-2} V

e. This emf causes a current of 75 A to flow in the ring. What is the force on the ring? *(5 pts.)*

$\mathbf{F} = \oint I\, d\mathbf{s} \times \mathbf{B} = 2\pi r I B = (2\pi)(0.086\ \mathrm{m})(75\ \mathrm{A})(0.19\ \mathrm{T}) = 7.7\ \mathrm{N}$

7.7 N

PROBLEM 3 Falling Bars



A conducting bar is sliding vertically with velocity v=3.7 m/s down frictionless conducting rails positioned perpendicular to a uniform magnetic field B=0.77 T, directed out of the page, as shown in the figure. The rails are 0.98 m apart and connected via a 1.49 Ω resistor.

a. Find the voltage across the resistor *(5 pts)*.

$$\Phi = BA \qquad \frac{d\Phi}{dt} = B\ell v \qquad \mathcal{E} = -\frac{d\Phi}{dt} = -(0.77\ \mathrm{T})(0.98\ \mathrm{m})(3.7\ \mathrm{m/s}) = -2.8\ \mathrm{V} \qquad V = |\mathcal{E}| = 2.8\ \mathrm{V}$$

2.8 V

b. What is the power dissipated in the resistor? *(5 pts)*

$P = V^2/R = (2.8\ \mathrm{V})^2/(1.49\ \Omega) = 5.2\ \mathrm{W}$

5.2 W

c. What is the magnitude and direction of the magnetic force on the bar? Please give the magnitude and direction (up, down, left, right, into the page, out of the page) of the magnetic force on the bar in the first and second boxes respectively. (5 pts)

$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B} \qquad I = V/R = -2.8\ \mathrm{V}/1.49\ \Omega = 1.9\ \mathrm{A} \quad |\mathbf{F}| = (1.9\ \mathrm{A})(0.98\ \mathrm{m})(0.77\ \mathrm{T}) = 1.4\ \mathrm{N}$

By Lenz's law, the force will be upwards, opposing the increased flux through the loop.

1.4 N	up

d. An identical apparatus is constructed and placed 4.8 cm in front of the apparatus pictured. The two bars are released at the same moment and fall next to each other through the field. What is the additional new force exerted by one bar on the other? In the second box, please specify whether this force is attractive or repulsive. *(5 pts)*

$$F = \frac{\mu_0 I_1 I_2}{2\pi a}\ell = \frac{(4\pi \times 10^{-7}\ \mathrm{Tm/A})(1.9\ \mathrm{A})^2}{(2\pi)(0.048\ \mathrm{m})}(0.98\ \mathrm{m}) = 1.4 \times 10^{-5}\ \mathrm{N}$$

Parallel currents attract!

1.4×10^{-5} N	Attractive

e. A galvanometer of internal resistance 64.5 Ω is connected in parallel with the resistor to measure the current. What is the ratio of galvanometer current to resistor current? *(5 pts)*

$$\frac{\text{current through galvanometer}}{\text{current through resistor}} = \frac{\mathbf{E}/R_{gal}}{\mathbf{E}/R} = \frac{R}{R_{gal}} = \frac{1.49\ \Omega}{64.5\ \Omega} = 2.3\%$$

2.3 %

PROBLEM 4

An electron moves in a circular loop in a region where the constant uniform magnetic field is **B** = 5.4 T (0.0 **i** + 0.0 **j** + 1.0 **k**). At time t=0 its velocity is **v**=3.0x10^{6} m/s (1.0 **i** + 0.0 **j** + 0.0 **k**).

a. At time t=0, what is the magnitude and direction (in terms of unit vectors) of the force on the electron? Please enter the magnitude in the first box and the direction in the second box (5 pts).

$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B} = (-1.6 \times 10^{-19}\,\text{C})(3 \times 10^{6}\,\text{m/s})(5.4\,\text{T})(\mathbf{i} \times \mathbf{k})$

$= (-2.6 \times 10^{-12}\,\text{N})(-\mathbf{j}) = 2.6 \times 10^{-12}\,\text{N}\ \mathbf{j}$

2.6 x 10^{-12} N	**j**

b. What is the radius of the electron's orbit? *(5 pts.)*

$$R = \frac{mv}{qB} = \frac{(9.109 \times 10^{-31}\,\text{kg})(3.0 \times 10^{6}\,\text{m/s})}{(1.6 \times 10^{-19}\,\text{C})(5.4\,\text{T})} = 3.2 \times 10^{-6}\,\text{m}$$

3.2 x 10^{-6} m

c. What is the angular speed of the electron? *(5 pts.)*

$$\omega = \frac{v}{R} = \frac{3.0 \times 10^{6}\,\text{m/s}}{3.2 \times 10^{-6}\,\text{m}} = 9.5 \times 10^{11}\,\text{/s}$$

9.5 x 10^{11} /s

d. Calculate the dipole moment magnitude and direction (in terms of unit vectors) for the current loop formed by the circulating electron. Please enter the magnitude in the first box and the direction in the second box *(5 pts.)*

$$I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{(1.602 \times 10^{-19}\,\text{C})(9.5 \times 10^{11}\,\text{/s})}{2\pi} = 2.4 \times 10^{-8}\,\text{A}$$

$|\mu| = IA = I(\pi R^2) = (2.4 \times 10^{-8}\,\text{A})(\pi)(3.2 \times 10^{-6}\,\text{m})^2 = 7.6 \times 10^{-19}\,\text{Am}^2$

Direction of dipole moment is anti-parallel to magnetic field

7.6 x 10^{-19} Am^{2}	**-k**

e. What is the torque on the current loop? *(5 pts.)*

$\tau = \mu \times \mathbf{B} = 0$ (μ and **B** are anti-parallel)

0 Nm