February 18, 2004 Physics 202

EXAM 1

Print your name and section **<u>clearly</u>** **on all** **<u>five</u>** **pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

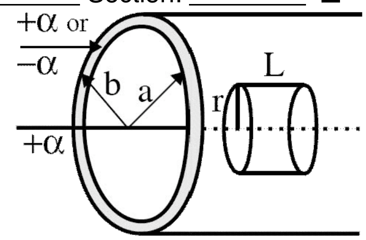
TOTAL: __________

SOLUTION KEY

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\, N^{-1}\, m^{-2}$
$k = 8.99 \times 10^{9}\ N\, m^2\, C^{-2}$
$e = 1.602 \times 10^{-19}\ C$

PROBLEM 1



A very long *conducting* tube (hollow cylinder) has an inner radius a and outer radius b and carries a charge per unit length of $+\alpha$ C/m. A line of charge $+\alpha$ C/m lies along the axis of the tube. No current is flowing. ***For all answers state why.***

a. What is the magnitude of the electric field at a radius $r < a$? *(5 pts.)*

Gauss: $E(2\pi rL) = \alpha L/\varepsilon_0 \Rightarrow E = \alpha/(2\pi\varepsilon_0 r)$

$\boxed{\alpha/(2\pi\varepsilon_0 r)}$

b. What is the magnitude of the electric field at a radius $a < r < b$? *(5 pts.)*

Field inside a conductor = 0

$\boxed{0}$

c. What is the magnitude of the electric field at a radius $r > b$? *(5 pts.)*

Gauss: $E(2\pi rL) = 2\alpha L/\varepsilon_0 \Rightarrow E = \alpha/(\pi\varepsilon_0 r)$

$\boxed{\alpha/(\pi\varepsilon_0 r)}$

d. The charge on the cylinder is changed to $-\alpha$. What is the magnitude of the electric field at radius $r < a$? *(5 pts.)*

Gauss: $E(2\pi rL) = \alpha L/\varepsilon_0 \Rightarrow E = \alpha/(2\pi\varepsilon_0 r)$

$\boxed{\alpha/(2\pi\varepsilon_0 r)}$

e. The charge on the cylinder remains $-\alpha$. What is the magnitude of the electric field at radius $r > b$? *(5 pts.)*

Gauss: $E(2\pi rL) = (\alpha-\alpha)L/\varepsilon_0 = 0 \Rightarrow E = 0$

$\boxed{0}$

PROBLEM 2

A metal (conducting) sphere of radius 0.160 m has a charge of 3.20 x 10^{-8} C.
a. What is the magnitude of the electric field on top of the surface of the sphere? *(5 pts.)*

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1^2} = \left(8.99\times10^9\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\right)\frac{\left(3.20\times10^{-8}\mathrm{C}\right)}{\left(0.160\mathrm{m}\right)^2} =$$

$$\boxed{1.12\mathrm{x}10^4\ \mathrm{N/C}}$$

b. What is the magnitude of the electric potential on top of the surface of the sphere? *(5 pts.)*

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1} = R_1E = \left(0.160\mathrm{m}\right)\left(1.12\mathrm{x}10^4\mathrm{N/C}\right) =$$

$$\boxed{1.80\mathrm{x}10^3\ \mathrm{V}}$$

This sphere is now connected by a long thin conducting wire to an initially uncharged second metal sphere of radius 0.0400 m that is many meters away from the first sphere. After electrostatic equilibrium has been reached *(for parts c,d,e)*:
c. What is the magnitude of the total charge on the second sphere? *(5 pts.)*

$$Q_1' + Q_2' = Q_1, \frac{Q_1'}{R_1} = \frac{Q_2'}{R_2} \Rightarrow Q_1' = \frac{Q_2'R_1}{R_2} \Rightarrow Q_2' + Q_2'\frac{R_1}{R_2} = Q_1 \Rightarrow Q_2'\left(1+\frac{R_1}{R_2}\right) = Q_1$$

$$Q_2' = \frac{Q_1}{\left(1+R_1/R_2\right)} = \frac{Q_1}{\left(1+\left(0.160\mathrm{m}\right)/\left(0.040\mathrm{m}\right)\right)} = \frac{1}{5}Q_1 = \frac{1}{5}\left(3.20\times10^{-8}\mathrm{C}\right) =$$

$$\boxed{0.64\mathrm{x}10^{-8}\mathrm{C}}$$

d. What is the magnitude of the electric potential on top of the surface of the second sphere? *(5 pts.)*

$$V_2' = \frac{1}{4\pi\varepsilon_0}\frac{Q_2'}{R_2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{1}{5}Q_1}{\frac{1}{4}R_1} = \frac{4}{5}V_1\ \left(\text{part b}\right) = \frac{4}{5}\left(1.80\times10^3\mathrm{V}\right) =$$

$$\boxed{1.44\ \mathrm{x}\ 10^3\ \mathrm{V}}$$

e. What is the magnitude of the electric field on top of the surface of the second sphere? *(5 pts.)*

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2'}{R_2^2} = \frac{V_2'}{R_2} = \frac{1.44\times10^3\mathrm{V}}{0.040\mathrm{m}} =$$

$$\boxed{3.60\mathrm{x}10^4\ \mathrm{V/m}}$$

PROBLEM 3

A parallel plate capacitor has plate area of 2.00×10^{-1} m^2 and plate separation of 1.00×10^{-2} m and is connected to a power supply that charges it to 3,000.0 V. It is then disconnected from the power supply. After that a sheet of insulating dielectric is inserted that completely fills the space between the plates. The voltage across the plates decreases to 1,000.0 V.

a. What is the charge on the plates before the power supply is disconnected? *(3 pts.)*

$C_0 = \varepsilon_0 A/d = (8.85\text{x}10^{-12}\text{ F/m})(2.00\text{x}10^{-1}\text{ m}^2)/(1.00\text{x}10^{-1}\text{ m}) = 1.77 \times 10^{-10}$ F *(first answer part b)*

$Q = C_0V_0 = (1.77\text{x}10^{-10}\text{ F})(3.00 \times 10^3\text{ V}) = 5.31 \times 10^{-7}$ C =

$\boxed{0.531\ \mu\text{C}}$

For the situation before the power supply is disconnected and the dielectric is inserted and for the situation after this is done, please calculate *(parts b,c,d)*:

b. The capacitance *(6 pts.)*

$\kappa = V_0/V$ =3,000.0 V/1,000.0 V = 3.00, $C = \kappa\, C_0 = 3.00(1.77\text{x}10^{-10}\text{ F}) = 5.31 \times 10^{-10}$ F

Before:	After:
177 pF	531 pF

c. The magnitude of the field inside the capacitor *(6 pts.)*

$E_0 = V_0/d = 3{,}000.\text{V}/1.00 \times 10^{-2}\text{ m} = 3.00 \times 10^5$ V/m

$E = V/d = 1{,}000.\text{V}/1.00\text{x}10^{-2}\text{ m} = 1.00\text{ x}10^5$ V/m

Before:	After:
0.300 MV/m	0.100 MV/m

d. The energy stored in the capacitor *(6 pts.)*

$U_0 = (1/2)C_0V_0^{\,2} = (1/2)(1.77 \times 10^{-10}\text{ F})(3{,}000.\text{V})^2 = 8.00 \times 10^{-4}$ J

$U = (1/2)CV^2 = (1/2)(5.31 \times 10^{-10}\text{ F})(1{,}000.\text{V})^2 = 2.70 \times 10^{-4}$ J

Before:	After:
0.800 mJ	0.270 mJ

After the power supply is disconnected and the dielectric is inserted, please calculate:

e. The permittivity of the dielectric and the induced charge on each face of the dielectric *(4 pts.)*

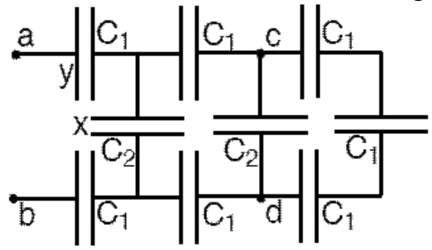
$$\varepsilon = \kappa\varepsilon_0 = (3.00)\left(8.85 \times 10^{-12}\,\text{C}^2/\text{Nm}^2\right) = 2.66 \times 10^{-11}\,\text{C}^2/\text{Nm}^2$$

$$Q_i = Q\left(1 - \frac{1}{\kappa}\right) = 5.31 \times 10^{-7}\text{C}\left(1 - \frac{1}{3.00}\right) = 3.54 \times 10^{-7}\text{C}$$

ε:	Q_i:
$2.66\text{x}10^{-11}\text{C}^2/\text{Nm}^2$	0.354μC

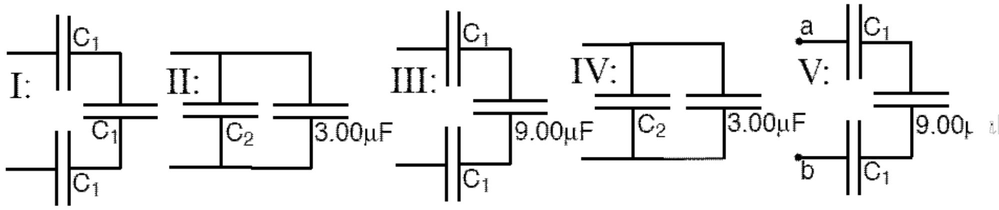
First Name: __________________________ Last Name: ________________________ Section: _________ ■

PROBLEM 4



A circuit contains 7 capacitors with C_1 = 9.00 μF (one of which is labeled “y”) and 2 capacitors with C_2 = 6.00 μF (one of which is labeled “x”).

a. What is the equivalent capacitance of this circuit between points *a* and *b*? *(10 pts.)*



$$\frac{1}{C_I}=\frac{3}{C_1}\Rightarrow C_I=\frac{9.00\mu F}{3}=3.00\mu F, C_{II}=3.00\mu F+6.00\mu F=9.00\mu F, \frac{1}{C_{III}}=\frac{2}{9.00\mu F}+\frac{1}{9.00\mu F}\Rightarrow$$

$$C_{III}=\frac{9.00\mu F}{3}=3.00\mu F, C_{IV}=C_{II}+6.00\mu F=3.00\mu F+6.00\mu F=9.00\mu F,$$

$$\frac{1}{C_V}=\frac{2}{C_1}+\frac{1}{9.00\mu F}=\frac{9.00\mu F}{3}=3.00\mu F$$

$\boxed{3.00\ \mu F}$

b. Compute the charge on capacitor “x” when V_{ab} = 900.0 V. *(5 pts.)*

$Q_X = C_2V_y = (6.00\times10^{-6}F)(300.\ V) =$

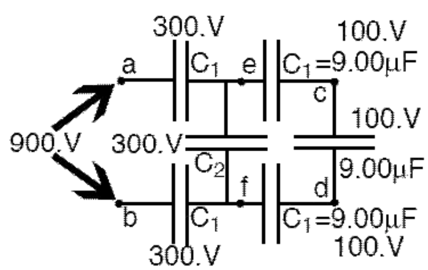
$\boxed{1.80\times10^{-3}\ C}$

c. Compute the charge on capacitor “y” when V_{ab} = 900.0 V. *(5 pts.)*

$Q_y = C_1V_y = (9.00\times10^{-6}F)(300.\ V) =$

$\boxed{2.70\times10^{-3}\ C}$

d. Compute the voltage V_{cd} when V_{ab} = 900.0 V. *(5 pts.)*



$V_{ec} = V_{cd} = V_{df}$ and $V_{ec} + V_{cd} + V_{df} = V_{ef} = 300.\ V \Rightarrow$
$3V_{cd} = 300.\ V \Rightarrow V_{cd} =$

$\boxed{100.\ V}$