FINAL EXAM

Print your name and section <u>**clearly**</u> **on all** <u>**nine**</u> **pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8.5" x 11" sheet and no other references. The exam lasts exactly two hours.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

Problem 5: __________

Problem 6: __________

Problem 7: __________

Problem 8: __________

TOTAL: __________

SOLUTION KEY

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$
$k = 8.99 \times 10^{9}\ N\ m^{-2}\ C^{-2}$
$e = 1.602 \times 10^{-19}\ C$
$m_e = 9.11 \times 10^{-31}\ kg$
$1\ eV = 1.602 \times 10^{-19}\ J$
$\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$
$1\ T = 1\ NA^{-1}m^{-1} = 10^4\ G$
$c = 2.998 \times 10^{8}\ m\ s^{-1}$

First Name: ____________________ Last Name:_____________________ Section: ________ ■

PROBLEM 1

A glass (n=1.50) thin lens has a flat back surface and its front surface is convex, with a radius of curvature of 10.0 cm.

a What is the focal length of the lens? *(5 pts.)*

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{10\,cm} - \frac{1}{\infty}\right) = \frac{0.5}{10\,cm} \Rightarrow f = 20\,cm$$

20 cm

b. An object is put 50.0 cm in front of the lens. How far is the image in **front** or in **back** *(circle one)* of the lens? *(5 pts.)*

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \Rightarrow \frac{1}{50\,cm} + \frac{1}{S'} = \frac{1}{20\,cm} \Rightarrow \frac{1}{S'} = \frac{1}{20\,cm} - \frac{1}{50\,cm} = \frac{1}{33\,cm}$$

33 cm

c. What is the magnification of the object in part b)? Is the image **inverted** or **erect** *(circle one)*? Is the image **real** or **virtual** *(circle one)*? *(5 pts.)*

$$m = \frac{-S'}{S} = \frac{-33\,cm}{50\,cm} = -0.67$$

-0.67

d. An object is put 15 cm in front of the lens. How far is the image in **front** or in **back** *(circle one)* of the lens? *(5 pts.)*

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \Rightarrow \frac{1}{15\,cm} + \frac{1}{S'} = \frac{1}{20\,cm} \Rightarrow \frac{1}{S'} = \frac{1}{20\,cm} - \frac{1}{15\,cm} = -\frac{1}{60\,cm} \Rightarrow \text{negative} \Rightarrow \text{in front}$$

60 cm

e. What is the magnification of the object in part d)? Is the image **inverted** or **erect** *(circle one)*? Is the image **real** or **virtual** *(circle one)*? *(5 pts.)*

$$m = \frac{-S'}{S} = \frac{-(-60\,cm)}{15\,cm} = +4.0$$

+4.0

PROBLEM 2

A slit 0.200 mm wide is illuminated by parallel rays of coherent light that has a wavelength of 500.0 nm. The diffraction pattern is observed on a screen that is 4.00 m from the slit. The intensity at the central maximum ($\theta = 0°$) is 5.00×10^{-6} W/m^2

a. What is the distance on the screen between the center of the central maximum to the first minimum? *(9 pts)*

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{500\times10^{-9}\,\text{m}}{0.200\times10^{-3}\,\text{m}} = 2.50\times10^{-3} \Rightarrow \theta_1 = 2.50\times10^{-3}\,\text{rad} \Rightarrow$$

$$y_1 = D\tan\theta_1 = 4.00\,\text{m}\times\tan\left(2.50\times10^{-3}\,\text{rad}\right) = 0.01\,\text{m}$$

10.0 mm

b. What is the total phase difference from the center of the central maximum to a point on the screen midway between the center of the central maximum and the first minimum? *(8 pts)*

$$\beta = \frac{2\pi a}{\lambda}\sin\left(\frac{\theta_1}{2}\right) = \frac{2\pi}{500.0\times10^{-9}\,m}\left(0.200\times10^{-3}\,m\right)\sin\left(1.25\times10^{-3}\,rad\right) = \pi$$

π

c. What is the intensity at the point on the screen midway between the center of the central maximum and the first minimum? *(8 pts)*

$$I = I_0\left[\frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}}\right]^2 = \left(5.00\times10^{-6}\,\frac{W}{m^2}\right)\left[\frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}}\right]^2 = \frac{4}{\pi^2}\left(5.00\times10^{-6}\,\frac{W}{m^2}\right) = 2.03\times10^{-6}\,\frac{W}{m^2}$$

$2.03\text{x}10^{-6}$ W/m^2

PROBLEM 3

A parallel plate capacitor is made from metal foil strips each 3.0 m long by 4.0 cm wide, separated by a polyethylene (dielectric constant = 2.3, dielectric strength = 7.5 x 10^8 V/m) sheet 2.0 x 10^{-5} m thick. We store 0.5 J of energy on the capacitor. (ε_0=8.854 x 10^{-12} F/m)

a What is the capacitance? *(5 pts.)*

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(2.3)\left(8.854\times10^{-12}\,\mathrm{F/m}\right)(3.0\mathrm{m}\times.040\mathrm{m})}{2.0\times10^{-5}\,\mathrm{m}} = 1.22\times10^{-7}\,\mathrm{F}$$

0.12 μF

b. What is the potential difference between the plates? *(5 pts.)*

$$U = \frac{1}{2}CV^2 \Rightarrow V = \sqrt{2U/C} = \sqrt{2(0.5\mathrm{J})/\left(1.22\times10^{-7}\,\mathrm{F}\right)} = 2860\mathrm{V}$$

2.9 kV

c. What is the stored charge? *(5 pts.)*

$Q=CV = (1.22 \times 10^{-7}\ \mathrm{F})(2860\ \mathrm{V}) = 3.4 \times 10^{-4}\ \mathrm{C}$

340 μC

d. What is the electric field strength? *(5 pts.)*

$E = V/d = (2860\ \mathrm{V})/(2.0 \times 10^{-5}\ \mathrm{m}) = 1.43 \times 10^{8}\ \mathrm{V/m}$

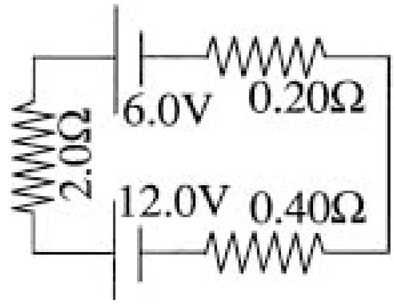
143 MV/m

e. If a safe capacitor can hold up to 10% of its breakdown voltage, is this capacitor safe when storing 0.5 J? *(5 pts.)*

(750 MV/m)/(143 MV/m) = 5.2 < 10

Not Safe

PROBLEM 4



A battery having an open circuit voltage of 12.0 V and an internal resistance of 0.40 Ω is connected (positive to positive) to a battery having an open circuit voltage of 6.0 V and an internal resistance of 0.2 Ω. The connecting leads contain a 2.0 resistor.

a. What is the current in the circuit? *(5 pts.)*

$12.0\ \text{V} - 6.0\ \text{V} = (2.0\ \Omega + 0.2\ \Omega + 0.4\ \Omega) \times I \Rightarrow 6.0\ \text{V} = 2.6\ \Omega \times I \Rightarrow I = 2.31\ \text{A}$

2.3 A

b. What is the power produced by the 12.0 V battery? *(5 pts.)*

$P = IV = 2.31\ \text{A} \times 12.0\ \text{V} = 27.7\ \text{W}$

28 W

c. What is the power dissipated in the 2.0 Ω resistor? *(5 pts.)*

$P = I^2R = (2.31\ \text{A})^2 \times 2.0\ \Omega = 10.7\ \text{W}$

11 W

d. What is the total power lost in the internal resistance of the two batteries?*(5 pts.)*

$P = I^2R = (2.31\ \text{A})^2 \times 0.40\ \Omega = 2.13\ \text{W}$ in 12.0 V Battery plus
$P = I^2R = (2.31\ \text{A})^2 \times 0.20\ \Omega = 1.07\ \text{W}$ in 6.0 V Battery give a total of 3.20 W

3.2 W

e. What is the charging power input to the 6.0 V battery? *(5 pts.)*

$27.7\ \text{W} - 10.7\ \text{W} - 3.2\ \text{W} = 13.8\ \text{W}$ or $P = IV = (2.3\ \text{A})(-6.0\ \text{V}) = -13.9\ \text{W}$ (- = power input)

14 W

PROBLEM 5

A solenoid is made of two coils wound on a 0.50 m long cardboard tube with cross sectional area $6.0x10^{-4}$ m^2. The primary coil has exactly 200 turns and a resistance of 3.0 Ω The secondary coil of exactly 1000 turns is wound closely on the first in the same direction, but is electrically insulated from it. A 12.0 V battery and a switch are connected to the first coil and there is a steady current after the switch is closed.

a. What is the magnetic flux in the solenoid when the steady current is established? *(6 pts.)*

$I = V/R = 12.0V/3.0 = 4.0A$, $B = \mu_0(n/L)I = (4\pi\times10^{-7}\ Tm/A)(200/0.50m)(4.0A) = 2.0\times10^{-3}T$

$\Phi = B_\perp A = 2.0\times10^{-3}T \times 6.0\times10^{-4}\ m^2 = 1.2\times10^{-6}\ Wb$

1.2×10^{-6} Wb

b. What is the magnitude of the average induced EMF in the second coil if the current in the primary coil falls to zero in $0.80x10^{-3}$ sec when the switch is opened? Is the direction of the EMF the **same** or opposite to that in the first coil (circle one)? *(7 pts.)*

$\varepsilon/turn = -\Delta\Phi/\Delta t = -(1.2\times10^{-6}\ Wb)/(8.0\times10^{-4}\ sec) = -1.5\times10^{-3}\ V/turn$,

$\varepsilon = turns\times\varepsilon/turn = -1.5\times10^{-3}\ V/turn \times 1000\ turns = -1.5\ V$ (- sign means opposing change!)

1.5 V

c. What is the mutual inductance of the two coils? *(5 pts.)*

$\varepsilon = M\times(\Delta I/\Delta t) \Rightarrow 1.5\ V = M(4.0\ A/8.0\times10^{-4}\ sec) \Rightarrow M = (1.5V \times 8.0\times10^{-4}sec)/4.0\ A = 3.0\times10^{-4}\ H$

0.30 mH

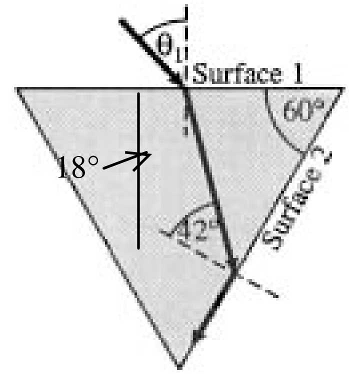
d. What is the mutual inductance if the tube is filled with iron of permeability 1500 times that of air? *(6 pts.)*

$\Phi' = 1500\ \Phi \Rightarrow \Delta\Phi' = 1500\ \Delta\Phi \Rightarrow \varepsilon' = 1500\ \varepsilon \Rightarrow M' = 1500\ M = 1500 \times 3.0\times10^{-4}\ H = 0.45\ H$

0.45 H

PROBLEM 6

A light beam of wavelength 550.0 nm traveling in air enters a solid prism at surface 1 and strikes surface 2 of the prism at an angle of 42.0° with respect to the perpendicular to the surface as shown. The light beam is totally internally reflected parallel to surface 2.



a. What is the index of refraction of the prism? *(5 pts.)*

$n \sin(42.0°) = \sin(90.0°)$

$n = 1/\sin(42°) = 1.49$

1.49

b. What is the speed of light in the prism? *(5 pts.)*

$v = c/n = (2.998 \times 10^8 \text{ m/s})/1.49 = 2.01 \times 10^8 \text{ m/s}$

2.01×10^8 m/s

c. What is the wavelength of the light in the prism? *(5 pts.)*

$\lambda_n = \lambda_0/n = 550.0 \text{ nm}/1.49 = 369 \text{ nm}$

369 nm

d. What is the frequency of the light in the prism? *(5 pts.)*

$f = c/\lambda_0 = (2.998 \times 10^8 \text{ m/s})/(550.0 \times 10^{-9}\text{m}) = 5.45 \times 10^{14} \text{ Hz}$

5.45×10^{14} Hz

e. What is the angle of incidence, θ_1 *in degrees*, of the light beam at surface 1 with respect to a line perpendicular to that surface? *(5 pts.)*

$18.0° = 60.0° - 42.0°$

$\sin(\theta_1) = n\sin(18.0°) = 1.49 \times 0.309 = 0.460 \Rightarrow \theta_1 = \sin^{-1}(0.460) = 27.4°$

27.4°

PROBLEM 7

A water drop (n=1.33) lies on a horizontal glass plate (n=1.52). The drop is thicker near its center than near the edge. At a horizontal distance 0.60 mm from the edge the drop is 0.673 μm thick.

a. At what wavelength between 400.0 nm and 700.0 nm does this part of the drop appear completely dark in reflected light? *(5 pts.)*

Because the water's refractive index is higher than air and lower than glass, upon reflection the phase changes by π at both the top and the bottom surface of the water layer. So destructive interference occurs when $2nt=(m+1/2)\lambda$ for integer m, or

$\lambda=2nt/(m+1/2)=(2)(1.33)(0.673\ \mu)/(m+1/2) = (1.5295\ \mu)/(m+1/2)$.

For m=0, λ= 3.58μ; for m=1, λ= 1.19μ; for m=2, λ=716 nm; for m=3, λ=511 nm; for m=4, λ=398n m. The value between 400nm and 700 nm is:

511 nm

b. At a horizontal distance 0.500 mm from the edge of the drop, because of constructive interference, the layer looks brightest when illuminated by λ = 432 nm light and also when illuminated by λ = 648 nm light. The thickness of the water layer at this point is the minimum thickness that satisfies these conditions. Find the thickness of the water layer at this position. *(10 pts.)*

Water's refractive index is higher than air and lower than glass, so upon reflection the phase changes by π at both the top and the bottom surface of the water layer. Therefore, the thin-film constructive interference condition is $2nt = m\lambda$ for thickness t and for n = 1:33.

The two visible wavelengths λ_1 = 432 nm and λ_2 = 648 nm satisfy $2nt = m_1\lambda_1$ and $2nt = m_2\lambda_2$. We can get a relationship between m_1 and m_2 by combining the above two equations:

$$2nt = m_1\lambda_1 \text{ and } 2nt = m_2\lambda_2 \Rightarrow m_1\lambda_1 = m_2\lambda_2 \Rightarrow \frac{m_1}{m_2} = \frac{\lambda_2}{\lambda_1} = \frac{648\text{ nm}}{432\text{ nm}} = 1.5.$$

The smallest integer m_1 and m_2 that have the ratio 1.5 are 3 and 2.

So $2nt = m_1\lambda_1 \Rightarrow t=m_1\lambda_1/(2n)=(3)(432\text{ nm})/(2\times1.33) = 487$ nm:

487 nm

c. If instead of glass the drop were on plastic with index of refraction n=1.22, at what wavelengths between 400.0 nm and 700.0 nm would the part of the drop with thickness 0.565 μm appear completely dark in reflected light? *(5 pts.)*

The phase changes by π at the first reflection and by zero at the second reflection, so destructive interference when $2nt=m\lambda$ for integer m, or

$\lambda=2nt/m=(2)(1.33)(0.565\ \mu)/m = (1.5029\mu)/m$.

For m=1, λ=1.50μ; for m=2, λ=751 nm; for m=3, λ=501 nm; for m=4, λ=376 nm. The value between 400nm and 700 nm is

501 nm

d. For the water drop on plastic, what is the wavelength *in the plastic* of the visible light (wavelength in air between 400 and 700 nm) that has the most transmission into the plastic? *(5 pts.)*

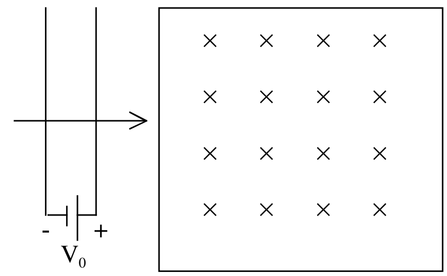
The condition for constructive interference in the plastic is the same as for destructive interference in reflection into the air, but keeping in mind that the $\lambda_{plastic} = \lambda_{air}/n_{plastic}$. So

$\lambda_{plastic} = \lambda_{air}/n_{plastic} = (501\text{ nm})(1.22)=$

410 nm

PROBLEM 8

The magnetic field of the Earth at a certain location is directed vertically downward (into the page as shown) and has a magnitude of 50.4 μT. A proton with mass $m_p = 1.673 \times 10^{-27}$ kg initially moving horizontally to the east at 1.23×10^6 m/s is accelerated across a voltage V_0 until it is moving horizontally toward the east as shown in this field with speed 7.15×10^6 ms^{-1}.



a. What is the magnitude of the voltage V_0? *(5 pts.)*

qV=change in kinetic energy = $m_p(v_f^2 - v_i^2)/2$, *so*

$$V = \frac{m_p(v_f^2 - v_i^2)}{2q} = \frac{\left(1.673\times10^{-27}\ \text{kg}\right)\left(\left(7.15\times10^{6}\ \text{m/s}\right)^2 - \left(1.23\times10^{6}\ \text{m/s}\right)^2\right)}{(2)\left(1.602\times10^{-19}\ \text{C}\right)} =$$

259 kV

b. What is the direction of the magnetic force that the field exerts on this charge (north, south, east, or west)? *(5 pts.)*

right hand rule: $q\vec{v}$ *points east,* $\vec{B}$ *points down, so* $q\vec{v}\times\vec{B}$ *points north*

north

c. What is the magnitude of the magnetic force the field exerts on this charge when it is moving at 7.15×10^6 m/s? *(5 pts.)*

$q\vec{v}$ *and* $\vec{B}$ *are perpendicular, so magnitude of force is*

$qvB = (1.602 \times 10^{-19}\ C)(7.15 \times 10^6\ m/s)(5.4 \times 10^{-5}\ T) =$

5.77×10^{-17} N

d. What is the radius of curvature of the trajectory followed by this proton? *(5 pts.)*

$$r = \frac{mv_f}{qB} = \frac{\left(1.673\times10^{-27}\ \text{kg}\right)\left(7.15\times10^{6}\ \text{m/s}\right)}{\left(1.602\times10^{-19}\ \text{C}\right)\left(5.04\times10^{-5}\ \text{T}\right)} =$$

1.48×10^3 m

e. The proton collides elastically with a second proton that is initially at rest. After the collision the trajectories of the two particles remain horizontal, and the radius of curvature of the first proton's trajectory has decreased by a factor of two compared to the value in part d. What is the kinetic energy in Joules of the second proton after the collision? *(5 pts.)*

Since $r \propto v$, *after the collision the kinetic energy of first particle is reduced by a factor of 4. So second proton has the kinetic energy* $\frac{3}{4}\left(\frac{1}{2}m_p v_f^2\right) = \frac{3}{4}\left(\frac{1}{2}\left(1.67\times10^{-27}\ \text{kg}\right)\left(7.15\times10^{6}\ \text{m/s}\right)^2\right) =$

3.21×10^{-14} J