# EXAM 2

**Print your name and section clearly on all five pages.** (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. **Your final answer must be placed in the box provided**. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. **Each problem is worth 25 points**. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 5" x 8" note card and no other references. The exam lasts exactly one hour.

*(Do not write below)*

**SCORE:**

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

**SOLUTION KEY**

**TOTAL:** __________

Possibly useful information:

$\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$

$k = 8.99 \times 10^9 Nm^2C^{-2}$

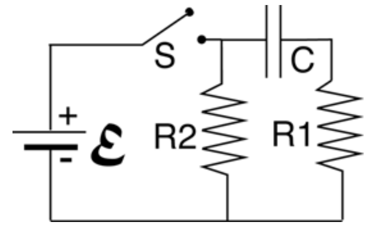
$\mu_0 = 4\pi \times 10^{-7} WbA^{-1}m^{-1}$

electron mass $m_e = 9.109 \times 10^{-31}$ kg

elementary charge $e = 1.602 \times 10^{-19}$ C

acceleration due to earth's gravity $g=9.80\ ms^{-2}$

**PROBLEM 1**




In the circuit shown $\mathcal{E}$ = 12.0 V, C = 6.0 μF, R1 = 4.0 Ω and R2 = 6.0 Ω. At t = 0, capacitor C is uncharged and the switch, S, which had been open, is suddenly closed.

a. What is the time constant τ for charging the capacitor? *(5 pts.)*

$\tau = R1 \times C = 4.0\ \Omega \times 6.0 \times 10^{-6}\ F = 2.4 \times 10^{-5}\ s$

| |
|---|
| 24 μs |

b. At t = 2.0τ, what is the voltage across the capacitor? *(5 pts.)*

$$V_C = \varepsilon\left(1 - e^{-t/\tau}\right) = 12V\left(1 - e^{-2.0}\right) = 10V$$

| |
|---|
| 10 V |

c. At t = 2.0τ, what is the voltage across resistor R1? *(5 pts.)*

$\varepsilon - V_C - V_{R1} = 0 \Rightarrow V_{R1} = \varepsilon - V_C = 12V - 10\ V = 2\ V$

| |
|---|
| 2V |

d. At t = 2.0τ, what is the voltage across resistor R2? *(5 pts.)*

$\varepsilon - V_{R2} = 0 \Rightarrow V_{R2} = \varepsilon = 12\ V$

| |
|---|
| 12 V |

e. At t = ∞, what are the voltages across resistors R1 and R2? *(5 pts.)*

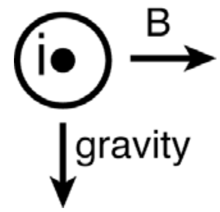
$V_C = \varepsilon\left(1 - e^{-\infty}\right) = 12V$ , $\varepsilon - V_C - V_{R1} = 0 \Rightarrow V_{R1} = \varepsilon - V_C = 12V - 12\ V = 0\ V$

$\varepsilon - V_{R2} = 0 \Rightarrow V_{R2} = \varepsilon = 12\ V$

| |
|---|
| R1: 0 V R2: 12 V |

## PROBLEM 2

A straight horizontal conducting wire of mass per length 46.6 g/m carries a current **i** = 28.0 A out of the page as shown. There is a uniform magnetic field, **B**, just barely sufficient to suspend it.




a. Show the direction of **B** on the figure. *(5 pts.)*

b. What is the magnitude of **B**? *(5 pts.)*

$$iLB = mg \Rightarrow B = \frac{(m/L)g}{i} = \frac{(46.6\times10^{-3}\,kg/m)(9.80\,m/s^2)}{28.0\,A} =$$

| |
|---|
| $1.63\times10^{-2}$ T |

c. The wire is removed and a 250 turn circular coil of radius 4.50 cm carrying a current of 13.0 A is placed so that the plane of the coil is parallel to **B**. What is the magnitude of the torque acting on the coil? *(5 pts.)*

$$\tau = \mu B = NIAB = NI(\pi r^2)B = (250)(13.0\text{ A})(\pi)(4.50\times10^{-2}\text{ m})^2(1.63\times10^{-2}\text{ T}) =$$

| |
|---|
| 0.337 Nm |

d. The coil is removed and a proton ($m_p = 1.67\times10^{-27}$ kg, $q_p = 1.60\times10^{-19}$ C) enters with a velocity $4.70\times10^6$ m/s perpendicular to **B**. What is the radius of its path? *(5 pts.)*

$$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\,kg)(4.70\times10^{6}\,m/s)}{(1.60\times10^{-19}\,C)(1.63\times10^{-2}\,T)} =$$

| |
|---|
| 3.01 m |

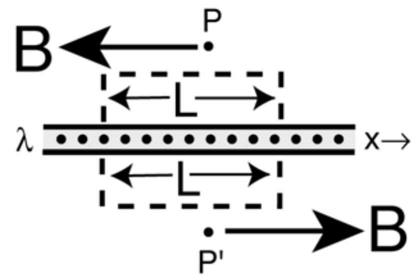
e. What is the period of the proton's circular motion? *(5 pts.)*

$$T = \frac{2\pi m}{qB} = \frac{(2\pi)(1.67\times10^{-27}\,kg)}{(1.60\times10^{-19}\,C)(1.63\times10^{-2}\,T)} = 4.02\times10^{-6}\,s$$

| |
|---|
| 4.02 μs |

**PROBLEM 3**

An infinite copper (resistivity 1.70 x $10^{-7}$ Ωm) sheet of thickness 1.27 mm with current per x-length λ = 26.0 A/m emerging perpendicularly out of the page is shown in cross section. A magnetic field **B** is produced in the air around the sheet.




a. Show the direction of **B** at points **P** and **P′** on the figure. *(5 pts.)*

b. What is the magnitude of **B** at **P** and **P′**? *(5 pts.)*

$$\oint B\,ds = 2BL = \mu_0 \lambda L \Rightarrow B = (1/2)\,\mu_0\lambda = (1/2)(4\pi\times10^{-7}\ \text{N/A}^2)(26.0\ \text{A/m}) =$$

| |
|---|
| $1.63\times10^{-5}$ T |

c. What is the voltage change per meter in the direction of the current? *(5 pts.)*

$$V = IR = I\left(\frac{\rho\ell}{tL}\right) \Rightarrow \frac{V}{\ell} = \frac{\lambda L \rho}{tL} = \frac{\lambda\rho}{t} = \frac{(26.0\ \text{A/m})(1.70\times10^{-7}\,\Omega\text{m})}{(1.27\times10^{-3}\,\text{m})} = 3.48\times10^{-3}\,\frac{\text{V}}{\text{m}} =$$

| |
|---|
| 3.48 mV/m |

d. What is the power dissipated per square meter of the copper sheet? *(5 pts.)*

$$P = IV = \lambda L\left(\frac{V}{\ell}\right)\ell \Rightarrow \frac{P}{\ell L} = \lambda\frac{V}{\ell} = \left(26.0\,\frac{\text{A}}{\text{m}}\right)\left(3.48\times10^{-3}\,\frac{\text{V}}{\text{m}}\right) = 9.05\times10^{-2}\,\frac{\text{W}}{\text{m}^2} =$$

| |
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| 90.5 mW/m$^2$ |

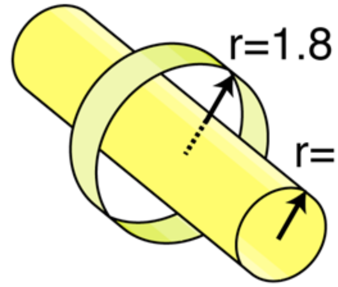
e. The current sheet is now placed in solid iron with magnetic permeability of 5,000$\mu_0$. What is the magnitude of the magnetic field **B** in the iron at **P** and **P'**? *(5 pts.)*

$$B = \frac{1}{2}\mu\lambda = \frac{1}{2}(5{,}000)\left(4\pi\times10^{-7}\,{}^{\text{N}}\!/_{\text{A}^2}\right)\left(26.0\,{}^{\text{A}}\!/_{\text{m}}\right) =$$

| |
|---|
| $8.17\times10^{-2}$ T |

**PROBLEM 4**

A long solenoid of radius 1.6 cm has 220 turns/cm and carries a current of 1.5 A. It is surrounded by a 120-turn coil of radius 1.8 cm and resistance 5.3 Ω centered on the axis of the solenoid and not connected to any source of EMF.




a. Find the magnetic field inside the solenoid. *(5 pts.)*

$B=\mu_0 in = (4\pi\times10^{-7}\ N/A^2)(1.5\ A)(220\ turns/10^{-2}\ m) = 4.1\times10^{-2}\ T$

| 0.041 T |
|---|

b. Find the magnetic flux inside the coil. *(5 pts.)*

$\Phi = BA = B\pi r_{sol}^2=(4.15\times10^{-2}\ T)(1.6\times10^{-2}\ m) = 3.34\times10^{-5}\ Wb$

| 33 μWb |
|---|

c. The current in the solenoid is reduced to 0 at a steady rate over 25 ms. Find the induced EMF in the coil during this time. *(5 pts.)*

For N turns in the coil, $\varepsilon = \frac{N\Delta\Phi}{\Delta t} = \frac{120\left(3.34\times10^{-5}\,\text{Wb}\right)}{25\times10^{-3}\,\text{s}} = 0.160\text{V}$

| 0.16 V |
|---|

d. Compute the current induced in the coil during the 25 ms. *(5 pts.)*

$$i = \frac{\varepsilon}{R} = \frac{0.160\text{V}}{5.30\Omega} = 0.0302\text{A}$$

| 30 mA |
|---|

e. Compute the energy dissipated in the coil during the 25 ms. *(5 pts.)*

$E=p\Delta t=i^2R\Delta t=(0.0302\ A)^2(5.3\Omega)(25\times10^{-3}\ s) = 1.21\times10^{-4} J$

| 0.12 mJ |
|---|