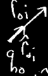
Electric Field

Force on a charge q_o at same coordinate $\vec{r}_o$ from charges q_i at $\vec{r}_i$

$$\vec{F}_o = q_o \vec{E}(\vec{r}_o)$$

$$\vec{E}(\vec{r}_0) = \sum_i k \frac{q_i \hat{r}_{oi}}{r_{oi}^2}$$



$|\hat{r}_{oi}| = 1$

$k = 8.9875 \times 10^9 \, Nm^2/C^2$

$= \frac{1}{4\pi\epsilon_o}$, ϵ_o = Permittivity of vacuum

$\epsilon_o = 8.8542 \times 10^{-12} \frac{C^2}{Nm^2}$

q is in Coulombs

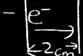
$\vec{F} = q\vec{E}$ if $q < 0$

$\vec{F}$ points opposite to $\vec{E}$

If $q > 0$

$\vec{F}$ points same as $\vec{E}$

Uniform $\vec{E}$ btw 2 oppositely charged parallel plates:



release electron from − plate travels to + plate in 1.5×10^{-8} sec

What is Electric Field?

$X - X_o = V_o t + \frac{1}{2} a t^2$

$V_0 = 0$ (start at rest)
$X - X_0 = 2\text{cm}$ (distance traveled)
$t = 1.5 \times 10^{-8}\,\text{s}$ (time)
$X - X_0 = \cancel{V_0 t}^{0} + \frac{1}{2} a t^2$
$a = 2(X - X_0)/t^2$
$F = ma \Rightarrow$ mass of electron $= m_e = 9.11 \times 10^{-31}\,\text{kg}$

$a = \frac{2(X - X_0)}{t^2} = \frac{2(0.02\,\text{m})}{(1.5 \times 10^{-8}\,\text{s})^2}$
$= 1.78 \times 10^{14}\,\text{m/s}^2$
$F = ma = m_e a = (9.11 \times 10^{-31}\,\text{kg})(1.78 \times 10^{14}\,\frac{\text{m}}{\text{s}^2})$
$= 1.62 \times 10^{-16}\,\text{N}$
$F = qE \Rightarrow E = F/q = F/e$
$= \frac{1.62 \times 10^{-16}\,\text{N}}{1.60 \times 10^{-19}\,\text{C}} = \boxed{1.01 \times 10^{3}\,\text{N/C}}$

$\vec{E} = \sum_i \frac{k q_i \hat{r}_{oi}}{r_{oi}^2}$
Define $\vec{E}_i = \frac{k q_i \hat{r}_{oi}}{r_{oi}^2}$
$\vec{E} = \sum_i \vec{E}_i$
field is sum of field contrib. of each charge ($\vec{E}_i$). Allowed because of Superposition

Continuous distribution of Charge

divide into discreet charges Δq_i each causing field $\Delta \vec{E}_{oi} = \frac{k \Delta q_i \hat{r}_{oi}}{r_{oi}^2}$

$$\vec{E}_o = k \sum_i \frac{\Delta q_i \hat{r}_{oi}}{r_{oi}^2} = \text{Total field}$$

Charges be smaller + multiply

$$\vec{E}_o = \lim_{\Delta q_i \to 0} k \sum_i \frac{\Delta q_i}{r_{oi}^2} \hat{r}_{oi} = k \int \frac{dq}{r_{oi}^2} \hat{r}_{oi}$$

Vector Integral.

Consider uniform charge distribution over volume V:

Charge density = Charge/Volume $\Leftrightarrow \rho = Q/V$

ρ has units of Coulombs/m^3

Charge Q spread over surface area A

Charge/Area $= \sigma = Q/A$ units $\frac{\text{Coulombs}}{m^2}$

Charge distributed uniformly along length ℓ along a line

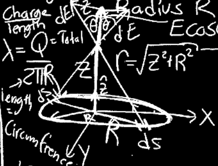
linear charge density: $\lambda = Q/\ell$ units $= \frac{\text{Coulombs}}{\text{meter}}$

For uniform charge in volume, or area or line

Volume: $dq = \rho\, dV$, Surface: $dq = \sigma\, dA$, line: $dq = \lambda\, dx$

Field of Ring of Charge Q along axis thru center

Charge/length $\lambda = \frac{Q}{2\pi R}$ = Total/Circumference



$dq_h = \lambda\, ds$

$$dq_h = \frac{Q}{2\pi R} ds$$

$$dE = k \frac{dq_h}{r^2} = \frac{kQ}{2\pi R} ds \frac{1}{z^2 + R^2}$$

In adding contributions from ds around ring, E components in x-y plane cancel each other:

only component that does not cancel = z component

$E_z = E\cos\theta$

Total Electric field = sum over all z components

$$\vec{E} = E_z \cdot \hat{z} \text{ or } E_z \cdot \hat{k}$$

$$E_z = E\cos\theta$$

$$dE \neq dE_z = \frac{kQ}{2\pi R} ds \frac{\cos\theta}{z^2 + R^2}$$

$$E_z = \int \frac{kQ}{2\pi R}\frac{\cos\theta}{z^2+R^2}\,ds$$

$$= \frac{kQ}{2\pi R}\frac{\cos\theta}{z^2+R^2}\int ds$$

$\int ds$ = circumfience of ring $= 2\pi R \Rightarrow$

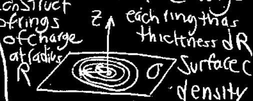
$$E_z = \frac{kQ\cos\theta}{z^2+R^2}, \text{ but } \cos\theta = \frac{z}{\sqrt{z^2+R^2}}$$

$$E_z = \frac{kQz}{(z^2+R^2)^{3/2}}, \quad \vec{E} = \frac{kQz}{(z^2+R^2)^{3/2}}\hat{z}$$

Check: $z=0$, $\vec{E}=0$ ✓

$z \gg R$: $\vec{E} = \frac{kQ}{R^2}\hat{z}$ ✓

Infinite sheet of Charge



Each Ring has an area = circumf × thickness $= 2\pi R\,dR$ and charge $= \sigma \cdot A = Q$

$$Q = \sigma \cdot 2\pi R\,dR$$

producing field

$$dE = \frac{k(2\pi R\,dR\,\sigma)z}{(z^2+R^2)^{3/2}}$$

$$E = \int_{R=0}^{R=\infty} \frac{k\, 2\pi R\, dR\, \sigma z}{(z^2+R^2)^{3/2}}$$

$$= 2\pi \sigma k z \int_0^{\infty} \frac{R\, dR}{(z^2+R^2)^{3/2}}$$

Look it up.

$$2\pi \sigma k z \left(\frac{1}{z}\right)$$

$$= 2\pi k \sigma$$

$$\vec{E} = 2\pi \sigma k\, \hat{z}$$

$$k = \frac{1}{4\pi \epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\, \hat{z}$$