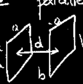
SPS tutoring
2321 Sterling
MW 9:55, 1:20, 2:25
TR 11
F 9:55, 1:20

Capacitance

1st parallel plates



$E = \sigma/\epsilon_0$ $\quad \sigma = \frac{\text{charge}}{\text{area}}$ on either plate

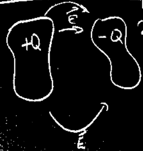
$$E = \frac{V_a - V_b}{d}$$

$$\Rightarrow \frac{\sigma}{\epsilon_0} = \frac{V_{ab}}{d}$$

charge on plate $= \sigma A$ ← plate area

$$Q = A\sigma = A\left(\frac{V_{ab}\epsilon_0}{d}\right)$$

$$Q = \left(\frac{A\epsilon_0}{d}\right)V_{ab} \Rightarrow C = \frac{\epsilon_0 A}{d} \text{ capacitance}$$



2 conductors

potential difference

$\Delta V \propto Q$

$\Delta V = Q/C$

$Q = C\Delta V$ capacitance

Capacitance relates
Q (charge) to ΔV (voltage difference)

depends on geometry.
but not on Q (or ΔV)

Unit of capacitance is Farad (F)

$$1\,F = 1\,C/V$$

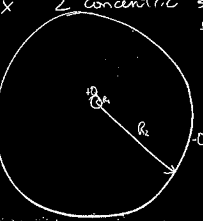
F is big unit
typical capacitors range from
10^{-12} F (pF) to 10^{-6} F (μF)

parallel plates

$$C = \epsilon_0 A/d.$$

Ex 2 concentric spheres.
find C

given $+Q$, $-Q$
find ΔV



Field between R_1 + R_2 is kQ/r^2
everywhere else, field = 0

spheres

$$\Delta V = \int_{R_1}^{R_2} \frac{KQ}{r^2} = \frac{KQ}{R_1} - \frac{KQ}{R_2}$$

$$C = \frac{Q}{\Delta V} = \frac{1}{(K/R_1 - K/R_2)}$$

can also say
total potential is sum of potentials from spheres 1 + 2

$$V_1 = \frac{KQ}{R_1} - \frac{KQ}{R_2} \Rightarrow V_1 - V_2 = \Delta V = KQ\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$V_2 = 0$$

As $R_2 \to 0$

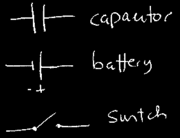
$$C \to \frac{R_1}{K}$$

"capacitance of sphere"

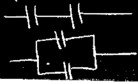
Combining capacitances
(in a circuit)

- series
- parallel

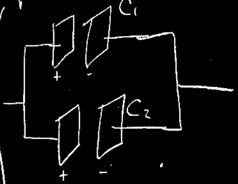
Symbols



capacitors



parallel



Voltage drop across $C_1 + C_2$ is same (ΔV)

need to find charge

$$Q_1 = C_1 \Delta V$$

$$Q_2 = C_2 \Delta V$$

$$\underbrace{Q_1 + Q_2}_{Q} = (C_1 + C_2)\Delta V$$

so $Q = C_{equiv} \Delta V$

$$\boxed{C_{equiv} = C_1 + C_2}$$

ΔV same across all C_i

$Q_1 = C_1 \Delta V$

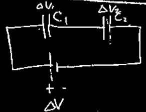
$Q_2 = C_2 \Delta V$

$Q_N = C_N \Delta V$

$$Q = \sum_{i=1}^{N} Q_i = \left(\sum_{i=1}^{N} C_N \right) \Delta V$$

$$C_{equiv} = \sum_{i=1}^{N} C_i$$

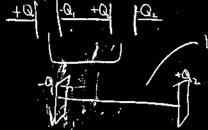
Capacitors in series:



$\Delta V_1 + \Delta V_2 = \Delta V$

charges on capacitors are equal

Charge on 2 capacitors is same



isolated, so $-Q_1 + Q_2 = 0$

$\Delta V_1 + \Delta V_2 = \Delta V$

$$\frac{Q}{C_1} + \frac{Q}{C_2} = \Delta V$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

for N capacitors in series,

$$\sum_{i=1}^{N} \Delta V_i = \Delta V$$

all charges same,

$$\sum_{i=1}^{N} \frac{Q}{C_i} = \Delta V$$

$$\Rightarrow \frac{1}{C_{equiv}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

$$\frac{1}{C_{equ}} = \frac{1}{8} + \frac{1}{8}$$
$$= \frac{1}{4}$$

if different combinations just do step by step