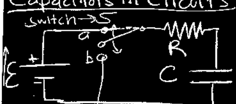
# Capacitors in Circuits




RC Circuit:

Switch at position (a)

Conservation of Energy:

Charge travel around circuit

In $\Delta t$, $Q = I\Delta t$ travels thru circuit

Work by EMF: $\Delta W = \mathcal{E}\Delta Q$

Must equal energy dissipated in R ($I^2 R\Delta t$) PLUS energy stored in Capacitor ($\frac{Q\Delta Q}{C}$)

$$\mathcal{E}\Delta Q = I^2 R\Delta t + \frac{Q\Delta Q}{C}$$

Divide by $\Delta t$

$$\mathcal{E}\frac{\Delta Q}{\Delta t} = I^2 R + \frac{Q\Delta Q}{C\Delta t}$$

$I = \Delta Q/\Delta t \Rightarrow$

$\mathcal{E} I = I^2 R + I Q/C \Rightarrow \div I$

If use loop theorem and remember $V_{cap} = Q/C$

$$\mathcal{E} - IR - Q/C = 0$$ around Loop

$\mathcal{E} = IR + Q/C$, $I = \frac{\Delta Q}{\Delta t}$

$\mathcal{E} = \frac{\Delta Q}{\Delta t}R + \frac{Q}{C}$, Use differentials

$\mathcal{E} = \frac{dQ}{dt}R + \frac{Q}{C}$ ÷ divide by R

$$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{Q}{RC} = -\frac{1}{RC}(Q - C\mathcal{E})$$

$$\frac{dQ}{Q - C\mathcal{E}} = -\frac{dt}{RC}$$

Integrate $q: 0 \to Q$, $t: 0 \to t$

$$\int_0^Q \frac{dq}{q - C\mathcal{E}} = -\int_0^t \frac{dt}{RC}$$

$$\ln\left(\frac{Q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

exponential both sides

$$\frac{Q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC} \Rightarrow Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

Let final charge $= Q_f = C\mathcal{E}$, $\boxed{Q = Q_f\left(1 - e^{-t/RC}\right)}$

Current $= i = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = \boxed{I_0 e^{-t/RC}}$

Let $I_0 = \frac{\mathcal{E}}{R}$ = initial current

at $t = 0$: $Q = 0$, $I = I_0 = \mathcal{E}/R$

at $t = \infty$: $Q = Q_f = C\mathcal{E}$, $I = 0$



Voltage across C

$Q = CV$

$V = Q/C$

"time to charge" given by product RC has dimensions of time = time for capacitor to reach $(1-e^{-1}) = 63\%$ of its final value

$$Q = C\mathcal{E}(1-e^{-RC/RC}) = C\mathcal{E}(1-e^{-1}) = 0.63\,C\mathcal{E}$$

for $t = RC$. Typical values, $C = 1.0\,\mu F$, $R = 2000\,\Omega$

$RC = 2\times10^{-3}$ secs

Back to circuit after long time: $t \gg RC$

Move switch ⓐ → ⓑ

No EMF in circuit


Loop:

$$IR + \frac{Q}{C} = 0$$

$$I = \frac{\Delta Q}{\Delta t} \Rightarrow R\frac{\Delta Q}{dt} + \frac{Q}{C} = 0$$

$$R\frac{dQ}{dt} = -\frac{Q}{C} \Rightarrow \frac{dQ}{dt} = -\frac{Q}{RC} \Rightarrow$$

$$\frac{dQ}{Q} = -\frac{dt}{RC} \Rightarrow$$ Integrate $q_i : Q_0 \to Q$

$Q_0 = C\mathcal{E}$ ← starting charge during $t: 0 \to t$ was Q at end of pt. 1

$$\int_{Q_0}^{Q} \frac{dq_i}{q_i} = -\frac{1}{RC}\int_0^t dt$$

exponentiate

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC} \Rightarrow \boxed{Q = Q_0 e^{-t/RC}}$$

$$i = \frac{dQ}{dt} = -\frac{Q_0}{RC} e^{-t/RC}$$

$$= I_0 e^{-t/RC}$$

$I_0 = Q_0/RC$

Initial Voltage across Cap $= \frac{Q_0}{C} = \mathcal{E}$

At Start (switch at ⓑ)

$Q_0 = C\mathcal{E}$, after time $t = RC$

$\rightarrow Q = Q_0 e^{-RC/RC}$

$= Q_0 e^{-1} = .37 Q_0$

time $t = RC$ is the time to reduce Charge to 37% of its original Value

Current $I = \frac{Q_0}{RC} e^{-t/RC}$

$Q_0 = C\mathcal{E}$; $I = \mathcal{E}/R \, e^{-t/RC}$

Current starts at $\mathcal{E}/R \rightarrow$ decreases

Energy: Charging (switch at ⓐ)

Battery delivers Power $\mathcal{E} i$

Energy dissipated in R: $i^2 R$

Energy stored in Cap: $iV = i\frac{Q}{C}$ (Cap Voltage)

$$\mathcal{E} i = i^2 R + i Q/C$$

Total Energy supplied (by Battery) $= \mathcal{E} Q_f$

Total Energy Stored in Cap $= \frac{1}{2} \mathcal{E} Q_f$ (final Volts on Cap)

Total energy dissipated in R is rest:

$= \mathcal{E} Q_f - \frac{1}{2}\mathcal{E} Q_f = \frac{1}{2}\mathcal{E} Q_f$

$\frac{1}{2}$ energy stored in Cap.

$\frac{1}{2}$ energy dissipated in R

How do we measure Currents + Voltages?

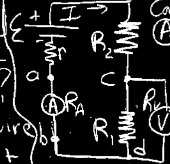
Measure current in wire

→ break wire + insert an Ammeter: —Ⓐ—

all current flows thru it.

→ Resistance of Ammeter must be small or changes current.

## Example Measuring Circuit



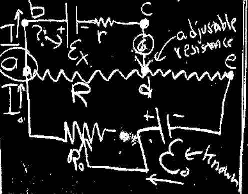

Condition on Ⓐ: $R_A \ll r + R_1 + R_2$

what about Voltages

Measure voltage across c, d with Voltmeter Ⓥ

$R_V \gg R_1$ → negligible current thru Ⓥ

How to measure EMF (not Voltage output)? Potentiometer + balance Voltages so no substantial current flows thru EMF



Ⓖ = Galvanometer
= very ± current meter
Loop theorem
$-\mathcal{E}_x - Ir + (I_0 - I)R = 0$
$I_0 - I$ = current in Big Resistor $R$
Junction Theorem at $\underline{\underline{a}}$
$I = \dfrac{I_0 R - \mathcal{E}_x}{R + r}$; Adjust $R$
till equals value $R_x$

$I_0 R_x = \mathcal{E}_x$