Magnetism

Charges in motion

Magnetic Field Vector $\vec{B}$ = Magnetic Induction or Field

Test charge at rest sees nothing

Test charge moves with velocity $\vec{V}$

Sideways (perpendicular) force acts on it

Vary direction of V, but not magnitude

Force remains perpendicular but magnitude changes

For a particular orientation of velocity $\vec{V}$ and $-\vec{V}$ find force is zero

All other directions for $\vec{V}$ have a force.

Direction where force is zero is unique and is defined to be direction of magnetic field vector. But $\vec{V}$ $\perp$ to $\vec{B}$ → V is MAX

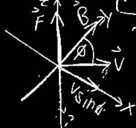
Magnitude of $\vec{B}$ defined in terms of max force $F_\perp$ when $\vec{V}$ is $\perp$ $\vec{B}$

Define $B = \frac{F_\perp}{q_0 V}$ for test charge q_0 moving w/ V

Velocity perpendicular to $\vec{B} = V_\perp$

$F = q_0 V_\perp B$

If ϕ is angle b/w V and B
$F = q_h V B \sin\phi$. If
$V \parallel B \Rightarrow \phi = 0 \Rightarrow F = 0$
$V \perp B \Rightarrow \phi = 90° \Rightarrow F = q_h V B$
Cross Product: $\boxed{\vec{F} = q_h \vec{V} \times \vec{B}}$



To Fully define Direction of Force ⇒ Specify sign:

Convention: $V_\perp$ along y
B along X, Force points along z axis
Right Hand Rule
(Also given by cross Prod)
Right Hand along $\vec{V}$
Fingers along $\vec{B}$
Force along Thumb.
For a positive charge
(negative: opposite to thumb)

Units: $B = \frac{F}{q_h V}$
$= \frac{\text{Newton-sec}}{\text{Coulomb}\cdot\text{meters}} = \frac{\text{Newtons}}{\text{Amp-Meter}}$
$= \text{Tesla} = 1\, N A^{-1} m^{-1}$
$= 10^4$ Gauss.

Force on current Carrying Conductor

Moving charges in $\vec{B}$

Wire length l, Current I perpendicular to $\vec{B}$

Current density $\vec{J} \perp \vec{B}$

Force on each electron:

$F = q_h V B \sin\phi$, $\phi = 90°$

$F = q_h V B$, $V = V_d$ = drift velocity

$F = q_h V_d B$, $V_d = J/ne$

n = # of conduction electrons per Volume

$q_h = e$ = electron charge

$$F = e\left(\frac{J}{ne}\right)B = \frac{JB}{n}$$

Wire X-sect area = A

Length = l, Volume = Al

Total Force on wire = Force on all electrons

electrons = $\frac{\#}{\text{Volume}}$ Volume

$n = \frac{\#}{\text{Volume}} \Rightarrow \#e = n \cdot \text{Vol} = nAl$

Total Force on wire = #e × force per e

$$= nAl \cdot \frac{JB}{n} = AlJB$$

Uniform current: $I = JA \Rightarrow J = \frac{I}{A}$

$$F = Al\frac{I}{A}B \Rightarrow \boxed{F = BIl}$$

If B is not perpendicular but angle θ then force on each electron $F = evB\sin\theta$ instead of $evB \Rightarrow$ on wire:

$$\boxed{F = BIl\sin\theta}$$

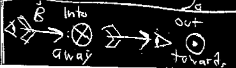
Vector $\vec{F} = I\vec{\ell} \times \vec{B}$

(ℓ points along wire in direction of current)

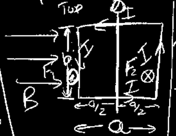
Arbitrarily Shaped Conductors:

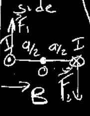
$$d\vec{F} = I\,d\vec{s} \times \vec{B}$$

Total Force $\vec{F} = I\int_a^b d\vec{s} \times \vec{B}$



Loop in magnetic field





Sides of length a are ∥ to $\vec{B}$ ⇒ no force on them

Mag: $F_1 = F_2 = IbB$

Pivot on axis O

Caculate Torque

$$\tau = F_1\frac{a}{2} + F_2\frac{a}{2} =$$

$$= IbB\left(\frac{a}{2}\right) + IbB\left(\frac{a}{2}\right)$$

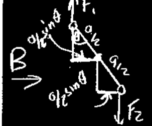
$$= IabB$$, but

ab = area of Loop:

$$\boxed{\tau = IAB}$$

Torque on Loop of area A, Current I in mag field B

Tilt the loop wrt $\vec{B}$:



$$\tau = F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta$$

$$= (IbB)\frac{a}{2}\sin\theta + (IbB)\frac{a}{2}\sin\theta$$

$$= IabB\sin\theta = IAB\sin\theta$$

Define area vector of Loop $\vec{A}$ has magnitude of area, direction perpendicular to plane of Loop:

$$\vec{\tau} = I\vec{A}\times\vec{B}$$

Product of current × area = magnetic moment = $\vec{\mu}$

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}, \quad \vec{\mu} = I\vec{A}$$

Besides Magnetic field Define Magnetic Flux...