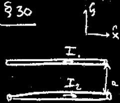
§30



Force between 2 current-carrying wires.

force on wire 1 due to B field from wire 2.

force on length l of 1 is

$$I_1 l \hat{x} \times \vec{B}_2$$

field due to wire 2.

$$\vec{B}_2 = \left(\frac{\mu_0 I_2}{2\pi a}\right)\hat{z}$$

Force $\vec{F}_1 = I_1 l \hat{x} \times \left(\frac{\mu_0 I_2}{2\pi a}\right)\hat{z}$

$$= -\hat{y} I_1 l \frac{\mu_0 I_2}{2\pi a}$$

parallel currents

$\Rightarrow$ wires attract
(force on wire 1 is towards wire 2)

magnetic forces on 2 wires are equal and opposite

$$\text{magnitude } \frac{\text{Force}}{\text{length}} = I_1 I_2 \frac{\mu_0}{2\pi a}$$

Define unit of currents

2 long parallel wires separated by 1m
when force is 2×10^{-7} N/m,
current is <u>defined</u> to be 1 A.

Use ampere to define Coulomb

if steady current of 1A
the amount of charge flowing
through cross section in 1s is 1C.

Amperes Law.

analogous to Gauss' law

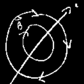
given closed loop

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

integrate along closed loop

I ← current enclosed by loop.

e.g. long straight wire



Choose circular path centered on wire.

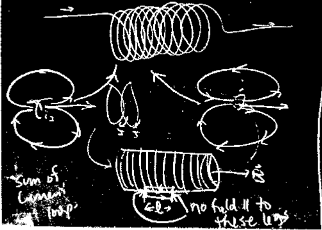
$$\oint \vec{B}\cdot d\vec{s} = 2\pi r B = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

(r = distance from wire)

Field of ideal solenoid

Solenoid long wire wound in form of helix



Outside solenoid, field is small

Calculate field inside use Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B l = \mu_0 \left[\frac{N}{l} l I \right]$$

N = # of turns in rectangle (length l)

I = current

define $n = N/l$ [# of turns/length]

$$B_{inside} = \mu_0 n I$$

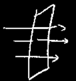
Magnetic flux through surface

define analogously to electric flux

$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

$d\vec{A}$ vector $\perp$ to surface
magnitude equal to area dA

$\vec{B}$ flux = 0



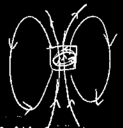
$\vec{B} \perp$ Area
flux = AB.

Gauss' law in magnetism

closed surface → $\oint\!\!\oint \vec{B} \cdot d\vec{A} = 0$

(recall "electric" Gauss' law: $\oint\!\!\oint \vec{E}\, d\vec{A} = \frac{Q}{\epsilon_0}$)

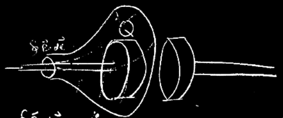
electric versus magnetic dipole:



far from source, fields are same
(but not same everywhere)

Maxwell's displacement current

Ambiguity in Ampere's law if time-dependent E fields.



$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i$ [yellow]

$\oint \vec{B}\cdot d\vec{\ell} = 0$ if red surface

Maxwell resolved problem by postulating

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Φ_E = electric flux through surface

define displacement current

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 (I + I_d)$$

Resolves inconsistency:

flux through red surface

$$\Phi_E = \oiint_{red} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\frac{d\Phi_E}{dt} = \frac{I}{\epsilon_0}$$

so $I_d = I$

$$\Rightarrow \oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

for both red and yellow surfaces.