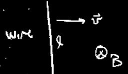
Motional EMF

EMF induced in a conductor moving through a constant magnetic field.



$\uparrow \vec{F}$ on +ve charges

$\downarrow \vec{F}$ on −ve charges

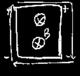
Lorentz force $\vec{F} = q\vec{v}\times\vec{B}$

charge builds up until electric force balances magnetic force

$$qE = qvB \quad \text{or} \quad E = vB$$

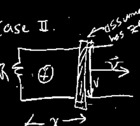
potential difference $\Delta V = E l = B l v$

Case I.



voltage around closed loop is zero

Case II.



move bar — set up current

voltage down bar

$\Delta V = Blv$

Show consistent with Faraday's law:

magnetic flux through loop $\Phi_B = B\ell x$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v$$

Find power dissipated in resistor

current $I = |\mathcal{E}|/R = B\ell v/R$

power $P = I^2R = B^2\ell^2v^2/R$

Force pulling bar related to power via

$$P = F_{app}v = B^2\ell^2v^2/R$$

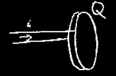
$$\Rightarrow F_{app} = B^2\ell^2v/R \checkmark$$

This agrees with another way to find F_{app}:

if current I flows through wire, then there is magnetic force $I\ell B$ opposing motion

to move wire, must apply $F_{app} = I\ell B = \left(\frac{B\ell v}{R}\right)\ell B = B^2\ell^2v/R \checkmark$

Clarification from last time



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$i = \frac{dQ}{dt}$$

$$\Phi_E = Q/\epsilon_0$$

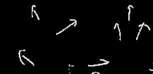
$$\Rightarrow \frac{d\Phi_E}{dt} = \frac{i}{\epsilon_0}$$

§ 30.8 Magnetism in matter

electrons in materials give rise to magnetic field

electron "spin"

"orbital" angular momentum



$$\vec{B} = \vec{B}_0 + \vec{B}_m$$

$\vec{B}_0$ ↑ field due to external currents

$\vec{B}_m$ ← field produced by material

define

$$\vec{B} = \mu_0 \vec{M}$$

$\vec{M}$ ↑ magnetization of material

define

$$\vec{H} = \vec{B}_0/\mu_0$$

(H = magnetic field strength)

if

$$\vec{M} = \chi \vec{H}$$

then can write

$$\vec{B} = \mu_m \vec{H}$$

where $\mu_m = \mu_0 (1 + \chi)$

or

$$\vec{B} = \frac{\mu_m}{\mu_0} \vec{B}_0$$

μ_m = magnetic permeability

§ 30 Faraday's Law of Induction

changing magnetic flux induces EMF

$$\underset{\text{EMF}}{\mathcal{E}} = -\frac{d\Phi_B}{dt}$$

[~~if~~ no charges]

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e.g. loop of area A in uniform field B. oriented at angle θ to normal

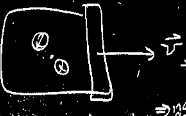
flux = $BA\cos\theta$

θ = angle between $\vec{B}$ and $\vec{A}$

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

§31.3 Lenz's law (the sign)

Induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux.



magnetic flux Φ_B is increasing in time

→ so induced current acts to decrease Φ_B

⇒ induced current flows counterclockwise

§ 31.4.

if electric fields are independent of time, then $\mathcal{E}$ is conservative

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$$

for closed loop if time independent $\oint \vec{E} \cdot d\vec{s} = 0$

when $\frac{d\Phi_B}{dt} \neq 0$, induced field is nonconservative

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

§31.5 Generators and motors.

generator — converts mechanical power to electrical power.

motor — converts electrical to mechanical power.