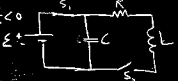
202 Exam
Wed Mar 16.
5:45 - 6:45 pm

§32 RLC circuit

$t<0$



$t=0$
open S_1 & close S_2

$t \geq 0$

find $Q(t)$ $\left[I(t) = \frac{dQ(t)}{dt}\right]$

$\sum$ voltage drops = 0

voltage across $C = Q/C$

voltage across $R = IR$

" " $L = L\,dI/dt$

note also $I = dQ/dt$

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

same eq'n as damped harmonic oscillator

if resistance large,

$Q(t) = Q_0 e^{-\Gamma t}$

$\frac{dQ(t)}{dt} = -\Gamma Q_0 e^{-\Gamma t}$

$\frac{d^2Q(t)}{dt^2} = \Gamma^2 Q_0 e^{-\Gamma t}$

plug into differential equation

$$L\Gamma^2 Q_0 e^{-\Gamma t} - R\Gamma Q_0 e^{-\Gamma t} + \frac{1}{C} Q_0 e^{-\Gamma t} = 0$$

$$\Gamma^2 - \frac{R}{L}\Gamma + \frac{1}{LC} = 0$$

$$\Gamma = \frac{1}{2}\frac{R}{L}\left(1 \pm \sqrt{1 - \frac{4L}{R^2 C}}\right)$$

real and solution is consistent as long as

$R^2 > 4L/C$

"critical resistance"

$R_c = \sqrt{4L/C}$

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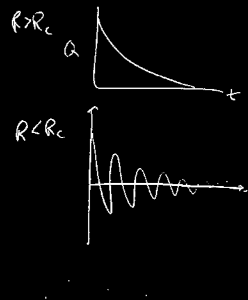
if $R < R_c$

result.

$Q(t) = Q(t=0)\,e^{-Rt/2L}\cos\omega t$

$\omega_0 = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$



§33 AC circuits

Consider sinusoidal drive

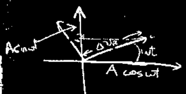
§33.2 resistor

AC voltage $\Delta v_R = \Delta V_{max}\sin\omega t$

current $i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R}\sin\omega t$

So $i_R = I_{max}\sin\omega t$ with $I_{max} = \frac{\Delta V_{max}}{R}$

$\Delta v_R = I_{max} R \sin\omega t$



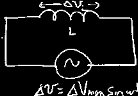
1) Define Rms current

$$I_{rms} = I_{max}/\sqrt{2}$$

$$P = I_{rms}^2 R$$

§33.3 Inductor in ac circuit

consider circuit:



$$\Delta v_L = \mathcal{E}_L = -L\frac{di}{dt}$$

$$\Delta V_{max}\sin\omega t - L\frac{di}{dt} = 0$$

$$di = \frac{\Delta V_{max}}{L}\sin\omega t\,dt$$

integrate:

$$i = -\frac{\Delta V_{max}}{\omega L}\cos\omega t\ (+C)$$

(C = 0 if any resistance at all)

Power dissipated in resistor

$$P(t) = i_R^2(t)R = I_{max}^2\sin^2\omega t\,R$$

average power $= R\langle I^2(t)\rangle = R\left(\frac{\omega}{2\pi}\right)\int^{2\pi/\omega} I_{max}^2\sin^2(\omega t) = R\left(\frac{\omega}{2\pi}\right)\left(\frac{1}{2}\right)\left(\frac{2\pi}{\omega}\right)I_{max}^2 = \frac{1}{2}I_{max}^2 R$

over one period $T = \frac{2\pi}{\omega}$

$$i = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

$$= -\frac{\Delta V_{max}}{\omega L}\left[-\sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$i(t) = \frac{\Delta V_{max}}{\omega L} \sin(\omega t - \pi/2)$$

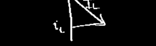
Current lags voltage

current lags voltage by $\pi/2$ (1/4 of cycle)

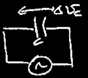
Note: max current $I_{max} = \frac{\Delta V_{max}}{\omega L}$

Define $\frac{\Delta V_{max}}{\Delta I_{max}} = \omega L \equiv X_L$ inductive reactance

$$I_{max} = \frac{\Delta V_{max}}{X_L}$$ [analogy with resistor: $I_{max} = \frac{\Delta V_{max}}{R}$]



§33.4 Capacitors in an AC circuit



$\Delta V = \Delta V_{max} \sin \omega t$

Kirchhoff's voltage law

$$\Delta V_C = \Delta V = \Delta V_{max} \sin \omega t$$

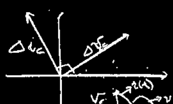
Since $q = C \Delta V_C$

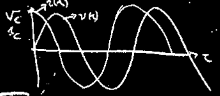
$$\therefore \quad q = C \Delta V_{max} \sin \omega t$$

current $i_C = \frac{dq}{dt} = C \Delta V_{max}\, \omega \cos \omega t = \omega C \Delta V \sin(\omega t + \pi/2)$

$$i_C = \omega C \Delta V \sin(\omega t + \pi/2)$$

current <u>leads</u> voltage (capacitor) by $\pi/2$





$$I_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

capacitive reactance $X_C = \frac{1}{\omega C}$

$$\Rightarrow I_{max} = \frac{\Delta V_{max}}{X_C}$$

$\Delta v = \Delta V_{max} \sin \omega t$

current $i = I_{max} \sin(\omega t - \phi)$

need to find I_{max} and ϕ