$\Delta v = \Delta V_{max} \sin \omega t$

current $i = I_{max} \sin(\omega t - \phi)$

find I_{max}, ϕ

$$\Delta v_R + \Delta v_L + \Delta v_C = \Delta v \quad \text{[Kirchhoff voltage]}$$

$$I_{max} R \sin(\omega t - \varphi) + I_{max} X_L \cos(\omega t - \varphi) - I_{max} X_C \cos(\omega t - \varphi) = \Delta V_{max} \sin \omega t$$

use $\sin(x-y) = \sin x \cos y - \cos x \sin y$ $\quad \cos(x-y) = \cos x \cos y + \sin x \sin y$

$$\Delta v_R = I_{max} R \sin(\omega t - \varphi) = \Delta V_R \sin(\omega t - \varphi) \quad [\Delta V_R = I_{max} R]$$

$$\Delta v_L = I_{max} X_L \sin(\omega t - \varphi + \pi/2) = \Delta V_L \cos(\omega t - \varphi) \quad [\Delta V_L = I_{max} X_L] \quad X_L = \omega L$$

$$\Delta v_C = I_{max} X_C \sin(\omega t - \varphi - \pi/2) = -\Delta V_C \cos(\omega t - \varphi) \quad [\Delta V_C = I_{max} X_C] \quad X_C = \frac{1}{\omega C}$$

$I_{max} R (\sin\omega t \cos\varphi - \cos\omega t \sin\varphi)$
$+ I_{max} X_L (\cos\omega t \cos\varphi + \sin\omega t \sin\varphi)$
$- I_{max} X_C (\cos\omega t \cos\varphi + \sin\omega t \sin\varphi)$
$= \Delta V_{max} \sin\omega t$

$\sin\omega t (I_{max} R \cos\varphi + I_{max} X_L \sin\varphi - I_{max} X_C \sin\varphi - \Delta V_{max})$
$+ \cos\omega t (-I_{max} R \sin\varphi + I_{max} X_L \cos\varphi - I_{max} X_C \cos\varphi) = 0$

true for all times ⇒

(1) $I_{max} R \cos\varphi + I_{max} X_L \sin\varphi - I_{max} X_C \sin\varphi - \Delta V_{max} = 0$ AND (2) $-I_{max} R \sin\varphi + I_{max} X_L \cos\varphi - I_{max} X_C \cos\varphi = 0$

Solve (1) + (2) ⇒

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\varphi = \frac{X_L - X_C}{R}$$

$X_L = \omega L \quad X_C = \frac{1}{\omega C}$

define impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$\Delta V_{max} = I_{max} Z$

Resonance in RLC circuit

calculate $I_{Rms} = \frac{\Delta V_{Rms}}{\sqrt{R^2+(X_L-X_C)^2}}$

max. I_{Rms} is $\frac{\Delta V_{Rms}}{R}$ $\begin{pmatrix} X_L = \omega L \\ X_C = 1/\omega C \end{pmatrix}$

when $X_L = X_C$

Resonance condition

$$\omega L = \frac{1}{\omega C}$$

$$\omega = 1/\sqrt{LC}$$

§ 33.6 Power in ac circuit

instantaneous power supplied is $I V$

$$\wp = i\Delta v = I_{max}\sin(\omega t-\varphi)\,\Delta V\sin\omega t$$
$$= I_{max}\,\Delta V_{max}\sin\omega t\sin(\omega t-\varphi)$$

time average of power:

$$\bar{\wp} = I_{max}\,\Delta V_{max}\sin\omega t\left[\sin\omega t\cos\varphi - \cos\omega t\sin\varphi\right]$$
$$= I_{max}\,\Delta V_{max}\left[\sin^2\omega t\cos\varphi - \sin\omega t\cos\omega t\sin\varphi\right]$$
$$= I_{max}\,\Delta V_{max}\left\{\left[\frac{1}{2} - \frac{1}{2}\sin 2\omega t\right]\cos\varphi - \frac{1}{2}\sin 2\omega t\sin\varphi\right\}$$

time average = 0   time average = 0

$$\bar{P} = \frac{I_{max}\,\Delta V_{max}}{2}\cos\varphi$$

$$\bar{P} = I_{rms}\,\Delta V_{rms}\cos\varphi$$

all power is dissipated in R:

max voltage drop across R is $I_{max} R$

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{\Delta V_{max}}{R}\,\frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$= \frac{\Delta V_{max}}{R}\cos\varphi$$

So $I_{rms} = \frac{\Delta V_{rms}}{R}\cos\varphi$

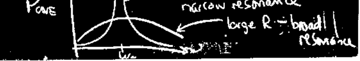
$$\Delta V_{rms}\cos\varphi = I_{rms} R$$

and $\underbrace{I_{rms}\,\Delta V_{rms}\cos\varphi}_{\bar{P}} = I_{rms}^2 R$

Calculate power as function of ω:

$$P_{AVE} = \frac{(\Delta V_{rms})^2 R}{Z^2} = \frac{(\Delta V_{rms})^2 R}{R^2+(X_L-X_C)^2}$$

$$P_{AVE} = \frac{(\Delta V_{rms})^2\,R\,\omega^2}{R^2\omega^2 + L^2(\omega^2-\omega_0)^2} \quad \left[\omega_0 = 1/\sqrt{LC}\right]$$

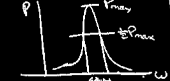


max power dissipated when $\omega = \omega_0$

$$\mathcal{P}_{max} = \frac{(\Delta V_{rms})^2 R \omega_0^2}{R^2 \omega_0^2} = \frac{(\Delta V_{rms})^2}{R}$$

Peak described by quality factor Q

$$Q = \frac{\omega_0}{\Delta\omega}$$



$\Delta\omega$ = width at half-power

$\Delta\omega = R/L$

so $Q = \omega_0 L/R$

high Q → response at resonant frequency

§33.8 transformers + power transmission

fixed power delivered to a load $\mathcal{P}_{AV}$

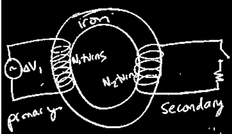
$$I_{rms} = \frac{\mathcal{P}_{AV}}{\Delta V_{rms}}$$

power dissipated in resistor R

$$= \left(\frac{\mathcal{P}_{AV}}{\Delta V_{rms}}\right)^2 R$$

higher ΔV_{rms} ⇒ lower loss

AC transformer converts high V to low V:



1st primary: Faraday:

$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt} \quad \Phi_B = \text{magnetic flux/turns}$$

secondary:

$$\Delta V_2 = -N_2 \frac{d\Phi_2}{dt}$$

but $\Phi_1 = \Phi_2$

$$\Rightarrow \Delta V_2 = \frac{N_2}{N_1} \Delta V_1$$

$N_2 > N_1 \Rightarrow \Delta V_2 > \Delta V_1$ "step-up transformer"

$N_2 < N_1 \Rightarrow \Delta V_2 < \Delta V_1$ "step-down transformer"