Mechanical Waves

2 Types: Transverse + Longitudinal

Transverse Wave:

Vibrate ~ $A \sin \omega t$

Amplitude A, frequency $f = \frac{\omega}{2\pi}$

Period $\tau = 1/f$

Distance btw. successive maxima = Wavelength = λ

Wave travels at constant speed c

advances distance of 1λ in one period τ. $c = \lambda/\tau$

$c = f\lambda$ = speed of wave

Longitudinal Wave

Long tube + plunger

Wavelength = distance between 2 compressions

$c = f\lambda$, $f = 1/\tau$

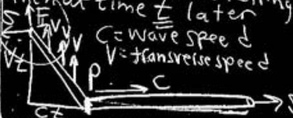
τ = time between compressions

Transverse waves on a string: speed

String under tension:

at $t=0$, apply constant transverse force $F_\perp$ to end of string

Then at time t later

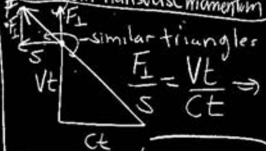


C = wave speed

V = transverse speed

Apply Impulse in transverse direction

(transverse force × time = Change in transverse momentum)



$$\frac{F_\perp}{S} = \frac{Vt}{Ct} \Rightarrow$$

$$F_\perp = S\frac{V}{C} \Rightarrow F_\perp t = S\frac{V}{C}t$$

Change in transverse Momentum

$$\Delta P_\perp = \mu C t V$$

$$\Rightarrow S\frac{V}{C}t = \mu C t V$$

$$C^2 = \frac{S}{\mu}$$

$$C = \sqrt{\frac{S}{\mu}}$$

Wave travels in x direction
moves up + down in y direction

At origin ($x = 0$). Assume
$y = A \sin \omega t = A \sin 2\pi f t$
travel to point x,
time to travel is $t = x/c$
Motion at x at time t is same
as that experienced at $x = 0$ at time $t - x/c$

Displacement $y(x,t)$
is found by replacing
t by $t - x/c$ from origin

$$y(x,t) = A \sin \omega \left(t - \frac{x}{c}\right)$$
$$= A \sin 2\pi f (t - x/c)$$

$c = f\lambda$, $f = 1/\tau$

$ft = \frac{t}{\tau}$, $c = f\lambda \Rightarrow \frac{2\pi}{\lambda} \cdot c = 2\pi f$

$$y(x,t) = A \sin 2\pi \left(\frac{t}{\tau} - \frac{x}{\lambda}\right)$$

Wavenumber

$k = \frac{2\pi}{\lambda}$; $\frac{2\pi}{\lambda} \cdot c = 2\pi f$

$\omega = ck$

$\frac{2\pi t}{\tau} = 2\pi f t = \omega t$

$2\pi x = kx$

$$y(x,t) = A \sin(kx - \omega t)$$

Displacement of particle from equilibrium as a function of x = "shape" of string at $t = 0$

$y = A\sin(-kx)$

$= -A\sin kx =$

$-A\sin 2\pi \frac{x}{\lambda}$

at $x = 0$ (origin)

$y = A\sin\omega t = A\sin\frac{2\pi t}{\tau}$

Wave traveling in $-x$ direction:

displacement at point x at time t is same as ~~displ~~ displacement at $x = 0$ at time $t + x/c$ later

replace t by $t + x/c$

$y = A\sin 2\pi f\left(t + \frac{x}{c}\right) =$

$A\sin 2\pi\left(\frac{t}{\tau} + \frac{x}{\lambda}\right) = A\sin(\omega t + kx)$

transverse speed of a particle in the string (not wave speed)

Wave speed $c = \lambda f = \omega/k$

particle speed $= \frac{\partial y}{\partial t}$ (x = const)

$y = A\sin(\omega t - kx)$

$v = \frac{\partial y}{\partial t} = \omega A\cos(\omega t - kx)$

acceleration (transverse)

$a = \frac{\partial^2 y}{\partial t^2} = \frac{\partial v}{\partial t} = -\omega^2 A\sin(\omega t - kx)$

Tansverse Velocity, accel:

$V_{max} = \omega A$, $a_{max} = \omega^2 A$

Wave transports energy because it moves string with mass/length = μ

$dm = \mu dx$. Each element undergoes Simple Harmonic Motion frequency ω, amplitude A.

Total Energy for SHM is $\frac{1}{2} k A^2$

k = spring constant

$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2$

Energy of piece of mass dm: $dE = \frac{1}{2} dm\, \omega^2 A^2$

(ωA = velocity)

$\Rightarrow dE = \frac{1}{2} \mu\, dx\, \omega^2 A^2$

so power

$$\frac{dE}{dt} = \frac{1}{2} \mu \frac{dx}{dt} \omega^2 A^2$$

$C = \frac{dx}{dt}$ = wave speed

$$\frac{dE}{dt} = \frac{1}{2} \mu C \omega^2 A^2$$

Power by 1-dimensional wave

for 3-D waves
rate of energy flow per unit area struck = average intensity

$$I_{av} = \frac{\text{avg Power}}{\text{unit area}}$$

$$= \frac{\text{average Energy}}{\text{Unit volume}} \times \text{Velocity} = \left(\frac{1}{2}\rho\omega^2 A^2\right) C$$

(density → ρ)