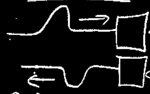
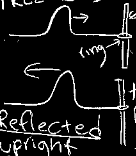
String: wave towards a fixed end:



Reflected Inverted

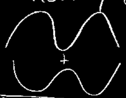
Wave towards a free end → No reaction force

FREE END



Reflected upright

If offset by ½ λ (180° out of phase) Cancel!



Destructive Interference

2 Sources of Sound cancel or add

2 waves of same frequency overlap

If they are "in phase":

= Sum of 2 waves Constructive Interference

Mathematically waves 1, 2:

$Y_1 = A\sin(kx - \omega t)$

$Y_2 = A\sin(kx - \omega t - \phi)$

Sum: $Y = Y_1 + Y_2 =$

$A\left(\sin(kx-\omega t) + \sin(kx - \omega t - \phi)\right)$

$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$

$a = kx - \omega t$, $b = kx - \omega t - \phi$

$$Y = 2A\cos\frac{\phi}{2}\sin\left(kx - \omega t - \frac{\phi}{2}\right)$$

$\cos\frac{\phi}{2} = \cos 0 = 1 \Rightarrow Y = 2A\sin(\ldots)$ = twice = Constructive Int.

$\cos\frac{\phi}{2} = \cos\frac{\pi}{2} = 0 \Rightarrow Y = 0$ destructive Int

Standing Waves on a string:

Train of waves strike fixed end → reflected

→ No movement: Nodes

Max movement: Antinodes

Distance between Nodes = Dist. btw. Antinodes $= \lambda/2$. Distance from Node to Antinode $= \lambda/4$

String fixed at both ends has a node at both ends

Since distance btw nodes $= \lambda/2 \Rightarrow$ wavelength

$\Rightarrow \frac{\lambda}{2} = L$

$\Rightarrow \lambda = L$

$\Rightarrow \frac{3\lambda}{2} = L$

Wavelengths on a string fixed both ends:
$\lambda = 2L, \frac{2L}{2}, \frac{2L}{3}, \ldots, \frac{2L}{n}$

$n = 1, 2, 3, \ldots$

Wave speed = C

$f = C/\lambda \Rightarrow$

$f = \frac{C}{2L}, \frac{2C}{2L}, \frac{3C}{2L}, \ldots, \frac{nC}{2L}$

Lowest frequency

$\frac{C}{2L}$ = fundamental frequency $= f_1$, and others

$= 2f_1, 3f_1, \ldots, nf_1$ are Overtones.

Overtones that are integral

Multiples of fundamental freq. f_1 are called harmonics

For a string $C = \sqrt{\frac{S}{\mu}}$, S = tension, $\mu = \frac{\text{mass}}{\text{length}}$

Longitudinal Standing Waves

Pressure nodes and antinodes

at a pressure node → pressure is constant, at antinode → pressure variation is a max

Pipe open at both ends → Antinode at both ends fundamental $f_1 = \frac{C}{2L}$

open:

| A N A | $\lambda_1 = 2L$

$f_1 = \frac{C}{2L}$

← L →

← $\lambda/2$ →

| A N A N A | $\lambda_2 = L$

$f_2 = \frac{C}{L} = 2f_1$

← λ →

Pipe closed at one end

"Stopped" pipe.

| A ← $\lambda/4$ → N | ← Node at closed end

antinode at open

Closed Pipe: $f_1 = \frac{C}{4L}$

½ of f_1 of open pipe (an octave lower)

Next frequency:

| A N A N |

← $\frac{3}{4}\lambda$ →

$\lambda_3 = \frac{4}{3}L$, $f_3 = \frac{3C}{4L} = 3f_1$

No f_2 possible

Different set of overtones or Harmonics

"Pitch" = Combination of fundamental frequency and harmonics

Beats: caused by interference between 2 waves with a similar frequency.

Add together ⇒ 3rd freq.

$Y = Y_1 + Y_2$

$Y_1 = A\cos 2\pi f_1 t$

$Y_2 = A\cos 2\pi f_2 t$

$Y = Y_1 + Y_2 = A(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$

$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$Y = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t \cdot \cos 2\pi\left(\frac{f_1+f_2}{2}\right)t$

Let $f' = \frac{f_1+f_2}{2}$ (avg of f_1, f_2)

$Y = A'\cos 2\pi f' t$ where

$A' = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$

Varying amplitude

If $f_1 \approx f_2 \Rightarrow f_1 - f_2$ very small, A' varys slowly

If A' Large → Sound is large and vice-versa

Max Amplitude A' for $\cos 2\pi\left(\frac{f_1-f_2}{2}\right) = 1$ or -1

occurs once per cycle ⇒ "beats" per second is twice frequency $\frac{f_1-f_2}{2}$ →

beats/second = difference in frequency

Hear beats up to 6 or 7 per second