Electromagnetic Waves

Changing AC Generator attached to antenna

old + new fields ⇒

E+B fields are perpendicular and alternate in direction

How do waves travel and how fast. EM Wave moves across rectangle abcd in xy plane

assume side ab in region where $E = B = 0$

Wave moves ⇒ magnetic flux through rectangular loop abcd

Changes, in time Δt, moves $\Delta X = V\Delta t$

V = wave speed, or equivalently rectangle

abcd → a'b'c'd': new area now has mag. flux

Change in flux = B field x area of dd'c'c

$$\Delta \Phi_B = B\Delta A = B y_0 \Delta X = B y_0 V \Delta t$$

y_0 = width ab = cd = a'b' = c'd'

Faraday: $\mathcal{E} = \dfrac{\Delta \Phi_B}{\Delta t} = \dfrac{B\Delta A}{\Delta t} = \dfrac{B y_0 V \Delta t}{\Delta t} = B y_0 V$

$$\boxed{\mathcal{E} = B y_0 V}$$

$\mathcal{E}$ around loop $\mathcal{E} = \mathcal{E}_{ab} + \mathcal{E}_{bc} + \mathcal{E}_{cd} + \mathcal{E}_{da}$

$\mathcal{E} = W/q_h = \vec{F}\cdot\vec{d}/q_h = \vec{E}\cdot\vec{d}$

$\mathcal{E}_{ab} = 0$ since $E(\text{region of } ab) = 0$

$\mathcal{E}_{bc} = \mathcal{E}_{da} = 0$ since $\vec{E} \perp bc$, $\vec{E} \perp da$

$\mathcal{E} = \mathcal{E}_{cd} = E\,Y_0$ (Y_0 = length cd)

E = magnitude of $\vec{E}$ along cd

$\mathcal{E} = E Y_0$ and $\mathcal{E} = B Y_0 V \Rightarrow$ $\boxed{E = VB}$

Electric flux thru X-Z plane:

Changing Electric flux thru new loop abcd

$= E \cdot \Delta A$, $\Delta A = Z_0 V \Delta t$, Z_0 = length ab = cd

(wave moves along X axis: $\Delta X = V\Delta t$)

Ampere's Law $\int B_{\parallel} dl = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

No current flowing $\Rightarrow I = 0$

$\int B_{\parallel} dl = \mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$

$\Delta \Phi_E = E \cdot \Delta A = E(Z_0 V \Delta t)$

$\int B_{\parallel} dl = \mu_0 \epsilon_0 \frac{E Z_0 V \Delta t}{\Delta t} = \mu_0 \epsilon_0 E Z_0 V$

$\boxed{\int B_{\parallel} dl = \mu_0 \epsilon_0 E Z_0 V}$

$\int B_{\parallel} dl = \sum B_{\parallel} \Delta l$ (4 sides of abcd)

but $B_{\parallel} \Delta l = 0$ for side ab since $B = 0$
and $= 0$ for sides bc, da because $B \perp$ side

Only remaining piece

$\sum B_{\parallel} \Delta l = B_{\parallel} (\Delta l \text{ for side cd})$
$= B z_0$, $\Rightarrow$ $\boxed{\int B_{\parallel} dl = B z_0}$

(But from Board above) $\Rightarrow$ $\int B_{\parallel} dl = \mu_0 \epsilon_0 E z_0 V$

$B z_0 = \mu_0 \epsilon_0 E z_0 V$

$\boxed{B = \mu_0 \epsilon_0 V E}$

Board above $\Rightarrow$ $\boxed{E = VB}$

$\Downarrow$

$B = \mu_0 \epsilon_0 V (VB)$

$1 = \mu_0 \epsilon_0 V^2 \Rightarrow$

$$V = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} C^2/Nm^2)(4\pi \times 10^{-7} Ns^2/C^2)}}$$

$V = 3.00 \times 10^8$ m/s
$= C$ = speed of light

Energy in a Wave

Start with energy in a E and B fields

Total energy density

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

For EM wave what are values of E + B?

$B = E/C$

$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\frac{E^2}{C^2}$

$C = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \Rightarrow$

$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\epsilon_0 \mu_0 E^2$

$\rightarrow$ $U = \epsilon_0 E^2$ $\quad E = CB \Rightarrow$

$U = \frac{1}{2}\epsilon_0 C^2 B^2 + \frac{B^2}{2\mu_0}$

$= \frac{1}{2}\epsilon_0 \frac{B^2}{\epsilon_0 \mu_0} + \frac{B^2}{2\mu_0} = \frac{B^2}{\mu_0} = U$

Energy density averaged over time

B_m = Max Value of B

E_m = Max Value of E

$B = B_m \cos(\omega t - kx)$

$E = E_m \cos(\omega t - kx)$

(modulo phase diff)

avg value of $\cos^2(\omega t - kx) = \frac{1}{2}$

average energy density

$U_{av} = \frac{\epsilon_0 E_m^2}{2} = \frac{B_m^2}{2\mu_0}$

Energy transported along x-axis is energy density * volume

$dV = A\,c\,dt$ (A = area, c dt = length into volume)

$dU = u\,dV = u\,A\,c\,dt$

$= \epsilon_0 E^2 A\,c\,dt$

Energy per time per area

$S = \frac{dU}{A\,dt} = \epsilon_0 c E^2$ since $1 = c^2 \epsilon_0 \mu_0$, $E = cB$

$S = \epsilon_0 c \frac{1}{c^2 \epsilon_0 \mu_0} E^2 = \frac{E^2}{c \mu_0} = \frac{EB}{\mu_0} = \frac{cB^2}{\mu_0}$

Units = energy flow

$= \frac{\text{Joules}}{s\,m^2} = W/m^2$

Vector: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

Poynting Vector (along any axis)