Homework X for December 8, 2014 – relativity

1. (Leighton) A particle of rest mass $M$ moving with speed $\beta_0 c$ collides inelastically with (sticks to) a stationary particle of rest mass $m$. Find the speed and rest mass of the final composite particle.

2. (Symon) Given two events $E_0$ and $E_1$ in $S$ for which

$$(r_1 - r_0)^2 - c^2(t - t_0)^2 > 0$$

there is a Lorentz transformation to a moving coordinate system $S^*$ in which $E_0$ and $E_1$ occur simultaneously. Let $E_0 = (0,0,0,0)$ and $E_1 = (x,0,0,t)$ and find the velocity of $S^*$ relative to $S$.

3. The Berkeley Bevatron was designed to have sufficient energy to produce antiprotons $\bar{p}$ in collisions with stationary protons by the reaction

$$p + p \rightarrow p + p + (p + \bar{p})$$

Both the $p$ and $\bar{p}$ have rest mass energy $M c^2 = 938$ MeV.

(a) What is the minimum energy (units of $M c^2$) in lab frame for the incident proton to be able to create antiprotons?

(b) If the experiment is done as a colliding beam experiment of two protons, each of energy $E$ but with equal and opposite momenta $\vec{p}$, what is the minimum energy $E$ to create antiprotons?

(c) If it is desired to have an energy of 1 TeV (= $10^{12}$ eV) available in the center of mass frame to create new particles, what is the required energy $E_i$ for a proton incident on a stationary proton?

4. An electron beam of 1 GeV (= $10^9$ eV) is made to orbit in a circle of radius 10 m by deflection in a uniform magnetic field $B$. What strength $B$ is required?
5. (Jackson) Use the relativistic velocity addition law to obtain the velocity of propagation $u$ of light in a moving liquid with frequency dependent refractive index $n(\omega)$ [the light travels with speed $u' = c/n(\omega')$ relative to the moving liquid]. Let the light travel parallel/antiparallel to the speed $v$ of the liquid and $\omega$ be the frequency of the light in the lab frame.

$$u = \frac{c}{n(\omega)} \pm v[1 - \frac{1}{n^2} + \frac{\omega}{n} \frac{dn}{d\omega}]$$

6. (Goldstein) A nucleus, originally at rest, decays radioactively by emitting an electron of momentum 1.73 MeV/c and at right angles to the direction of the electron a neutrino with momentum 1.00 MeV/c. Let the mass of the residual nucleus be $3.90 \times 10^{-25}$ kg = 220,000 MeV/c$^2$

   (a) In what direction does the nucleus recoil and what is its momentum?

   (b) The rest mass of the electron is 0.511 MeV and that of the neutrino is essentially zero. What are the total energies carried away by the electron (include rest mass), neutrino, and kinetic energy of the recoiling nucleus?

7. In an elastic collision of two equal rest-masses $M$, initially mass 2 is at rest at the origin $r = 0$ and mass 1 is incident from the left, along the $x$-axis with momentum $p = p\hat{x}$. Treat the final case in which the outgoing masses move at angles $\pm \vartheta/2$ relative to the $\hat{x}$-axis.

   (a) If the motion is nonrelativistic, give an explicit value for the angle $\vartheta$ between the outgoing masses.

   (b) If the motion is relativistic, give an expression for the angle between the outgoing masses and evaluate it in the limit $p >> Mc$. 

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