

Dispersion of the neutron resonance in cuprate superconductors

Andrey V. Chubukov,^{1,2} Boldizsár Jankó,^{1,3} and Oleg Tchernyshyov^{1,4}

¹Division of Materials Sciences, Argonne National Laboratory, Argonne, Illinois 60439

²Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

³Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556-5670

⁴School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540

(Received 7 December 2000; published 11 April 2001)

We argue that recently measured downward dispersion of the neutron resonance peak in cuprate superconductors is naturally explained if the resonance is viewed as a spin-1 collective mode in a d -wave superconductor. The reduction of the resonant frequency away from the antiferromagnetic wave vector is a direct consequence of the momentum dependence of the d -wave superconducting gap. When the magnetic correlation length becomes large, the dispersion should become magnonlike, i.e., curve upwards from (π, π) .

DOI: 10.1103/PhysRevB.63.180507

PACS number(s): 74.72.-h, 74.20.Fg, 74.25.-q, 71.10.Ca

Superconductivity and antiferromagnetism are two major phases of the high- T_c cuprates. When viewed separately, the corresponding ground states appear to be quite conventional: the parent compounds of high- T_c cuprates [such as La_2CuO_4 (Ref. 1)] are exemplary Heisenberg antiferromagnets, while overdoped cuprates, in many respects, resemble BCS-type d -wave superconductors.² However, full understanding of the interplay between the two phenomena is sorely lacking. Although most experts agree that antiferromagnetism is the ultimate cause of high- T_c superconductivity, the intermediate steps are not yet clear.³

Perhaps the strongest experimental indication of the interplay between antiferromagnetism and superconductivity in cuprates is the discovery of strong inelastic neutron scattering deep in the superconducting (SC) phase of materials with the highest T_c . The most intense scattering at $T \ll T_c$ has been detected in YBCO (Ref. 4) and Bi2212 (Ref. 5) near the antiferromagnetic (AF) wave vector $\mathbf{q} = \mathbf{Q} = (\pi, \pi)$ and energy $\Omega_{\mathbf{Q}} \approx 40$ meV. Experiments conducted with polarized neutrons⁶ indicate that the resonant scattering is due to electron spins.

It is tempting to interpret the resonance as a magnon in a disordered Néel state, which becomes more visible in a superconductor. However, a magnon frequency $\Omega_{\mathbf{q}}^2 = c^2(\xi^{-2} + |\mathbf{q} - \mathbf{Q}|^2) = \Omega_{\mathbf{Q}}^2 + c^2|\mathbf{q} - \mathbf{Q}|^2$, where c is a spin-wave velocity, *increases* away from the AF wave vector $\mathbf{Q} = (\pi, \pi)$.⁷ Meanwhile, recent experiments on YBCO (Refs. 8 and 9) have revealed the opposite trend: the resonance energy *decreases* away from (π, π) [see Fig. 1(a)]. In a disordered antiferromagnet, this can only be the case if spin response in the normal state is incommensurate.¹⁰ However, the data suggest⁹ that, unlike in 214 materials,¹¹ the normal-state spin response in YBCO is commensurate.

On second thought, the best superconductors of the cuprate family differ significantly from their parent AF compounds. They have a large, Luttinger-like Fermi surface in the normal state, and fermionic quasiparticles in the SC state, as shown by angle-resolved photoemission spectroscopy (ARPES).¹² Several authors have demonstrated that in this situation, an attractive exchange force in the d -wave superconducting state binds an electron and a hole into a pair with

total spin 1.¹³⁻¹⁹ The bound state is seen as a resonance in spin response. The gross features of the neutron resonance, such as a nonmonotonic variation of $\Omega_{\mathbf{Q}}$ with doping and persistence of the resonance in the pseudogap phase can be understood if the resonance peak is viewed as a collective mode.^{14,15,17,18}

The frequency of the collective mode is proportional to the static part of the inverse spin susceptibility $(\xi^{-2} + |\mathbf{q} - \mathbf{Q}|^2)^{1/2}$ and hence formally resembles that of a magnon in a disordered antiferromagnet. However, the velocity c is not constant but depends rather strongly on the wave vector \mathbf{q} . This dependence can be understood by noting that the bound state necessarily resides below the two-particle continuum [Fig. 1(c)]. For \mathbf{q} along a zone diagonal, the continuum starts at $E_{\min}(\mathbf{q}) = |\Delta(\mathbf{k})| + |\Delta(\mathbf{k} + \mathbf{q})|$, where both momenta \mathbf{k} and $\mathbf{k} + \mathbf{q}$ are at the Fermi surface, Fig. 1(b). As \mathbf{q} moves away from (π, π) , \mathbf{k} and $\mathbf{k} + \mathbf{q}$ shift along the Fermi surface towards the nodal points and the bound state inside the gap is pushed to lower energies. For $\mathbf{q} = \mathbf{q}_0$ connecting the nodal points, the two-particle spectrum is gapless, $E_{\min}(\mathbf{q}_0) = 0$, so that the energy of the resonance must vanish (along with its strength). Obviously then, $\Omega_{\mathbf{q}}$ should decrease away from (π, π) , at least near \mathbf{q}_0 .

The above qualitative argument is quite robust and is based on two premises only: (i) the Fermi surface contains

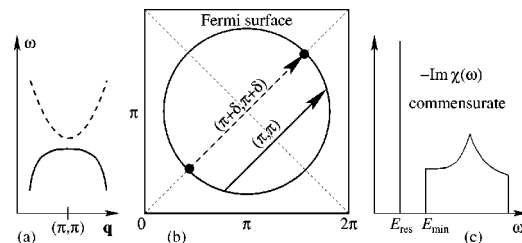


FIG. 1. (a) A sketch of the resonance dispersion: experimental (solid line), a massive magnon in the disordered Néel state (dashed line). (b) Approximate shape of the Fermi surface in Bi2212. Solid dots are nodal points, where the fermions are gapless. The continuum of two-particle excitations at the corresponding wave vector $\mathbf{q}_0 = (\pi + \delta, \pi + \delta)$ (dashed arrow) starts at zero frequency. For lattice momentum $\mathbf{Q} = (\pi, \pi)$ (solid arrow) the continuum starts at a finite energy. (c) Spin susceptibility at the wave vector (π, π) .

hot spots (i.e., points connected by the AF wave vector \mathbf{Q}) and (ii) the superconducting gap has the $d_{x^2-y^2}$ symmetry. A more quantitative description of the resonance requires further assumptions. Most calculations of a spin response in a d -wave superconductor start with a model of free fermions on a square lattice, add an exchange interaction and employ the random-phase approximation (RPA) to compute spin susceptibility.^{13,14,16-19} These calculations do yield a downward dispersion of the resonance under favorable circumstances.¹⁶⁻¹⁹ Unfortunately, such calculations must be done numerically providing somewhat limited insight. Here we present an alternative, analytical approach to the problem. It is based on the observation that the behavior of a collective mode is determined largely by low-energy fermion degrees of freedom and is therefore insensitive to the physics at high energies.

Our point of departure is a macroscopic spin-fermion model that describes low-energy fermionic quasiparticles (with a Fermi surface inferred from ARPES) interacting with collective spin fluctuations.²⁰ This model is viewed as a low-energy version of lattice, Hubbard-type models and is described by the effective action²⁰ which involves a bare fermion propagator $G_0^{-1}(\mathbf{k}, \omega) = \omega - \mathbf{v}_\mathbf{k} \cdot (\mathbf{k} - \mathbf{k}_F)$, a bare spin susceptibility $\chi_0(\mathbf{q}, \Omega)$ which is produced by high-energy fermions and is an input for low-energy theory and a spin-fermion coupling g . We assume that nothing special happens at high frequencies in which case $\chi_0(\mathbf{q}, \Omega)$ has an Ornstein-Zernike form: $\chi_0(\mathbf{q}, \Omega) = \chi_0 / [\xi^{-2} + (\mathbf{q} - \mathbf{Q})^2 - \Omega^2/v_s^2]$.

Previous studies of the model have focused on renormalization of the fermionic dispersion by spin-fermion interaction, and on the form of the full $\chi(\mathbf{Q}, \Omega)$.¹⁵ Here we consider the dynamical susceptibility at $\mathbf{q} \neq \mathbf{Q}$. To avoid unnecessary complications, we restrict calculations to momenta along the zone diagonal, $\mathbf{q} = (q, q)$. We also neglect strong coupling effects, assuming for simplicity that superconductivity is described by a d -wave version of the BCS theory. We have verified that strong-coupling effects (which modify fermionic propagator at low energies) result in quantitative, but not qualitative changes.

The full dynamical susceptibility $\chi(\mathbf{q}, \Omega)$ differs from $\chi_0(\mathbf{q}, \Omega)$ due to a bosonic self-energy $\Pi(\mathbf{q}, \Omega)$:

$$\chi(\mathbf{q}, \Omega) = \chi_0 / [\xi^{-2} + (\mathbf{q} - \mathbf{Q})^2 - \Omega^2/v_s^2 - \Pi(\mathbf{q}, \Omega)]. \quad (1)$$

The static part of $\Pi(\mathbf{q}, \Omega)$ (the contribution of the low-energy fermions to the inverse correlation length $1/\xi$) is small and, in fact, vanishes for linearized fermion dispersion. On the other hand, the frequency-dependent part $\Pi(\mathbf{q}, \Omega)$ is substantially nonzero in the normal state. This is related to the fact that, for a Fermi surface with hot spots (as in YBCO and Bi2212) a low-frequency spin excitation with momentum $\mathbf{q} \approx \mathbf{Q}$ can decay into two fermions at the Fermi surface (Fig. 1). This gives rise to a universal relaxational term in $\Pi(\mathbf{q}, \Omega)$:

$$\Pi(\mathbf{q}, \Omega) \approx i \operatorname{Im} \Pi(\mathbf{q}, \Omega) = i |\Omega| \gamma_\mathbf{q}. \quad (2)$$

At small frequencies, $\Pi(\mathbf{q}, \Omega) \propto |\Omega|$ is much larger than the bare $\mathcal{O}(\Omega^2)$ in the susceptibility. It thus fully determines spin dynamics at low energies. The prefactor $\gamma_\mathbf{q}$ is obtained

by calculating the imaginary part of the particle-hole bubble. At the antiferromagnetic momentum, $\gamma_\mathbf{Q} = 2\bar{g}/(\pi v_x v_y)$, where $\bar{g} = g^2 \chi_0$ is the effective spin-fermion coupling, and $\mathbf{v} = (v_x, v_y)$ is a Fermi velocity at the hot spot \mathbf{k}_{hs} , whose components are defined as $\epsilon_\mathbf{k} = v_x \tilde{k}_x + v_y \tilde{k}_y$, $\epsilon_{\mathbf{k}+\mathbf{Q}} = -v_x \tilde{k}_x + v_y \tilde{k}_y$, where $\tilde{\mathbf{k}} = \mathbf{k} - \mathbf{k}_{\text{hs}}$.¹⁵ At $\mathbf{q} = \mathbf{q}_0$ which connects the nodes, v_y vanishes and $\gamma_\mathbf{q}$ formally diverges. We have verified that at this momentum, $\Pi(\mathbf{q}_0, \Omega) \propto \sqrt{\Omega}$. In between \mathbf{Q} and \mathbf{q}_0 , the behavior of $\gamma_\mathbf{q}$ is somewhat involved: it first drops at the lowest frequencies due to a reduction in the number of the scattering channels from 8 to 2 (this is because q_i and $2\pi - q_i$ are no longer identical points, and then slowly increases on approaching \mathbf{q}_0 . The finite drop in $\gamma_\mathbf{q}$ is irrelevant for our consideration, and for simplicity we just assume below that for each \mathbf{q} , there are two ‘‘hot spots’’ $k = k_{k_s}(\mathbf{q})$ in the Brillouin zone, which account for the bosonic damping at the lowest frequencies.

Equations (1) and (2) imply that in the normal state

$$\operatorname{Im} \chi(\mathbf{q}, \Omega) \propto x / [(1 + \tilde{q}^2 \xi^2)^2 + x^2], \quad (3)$$

where $x = \Omega / \gamma \xi^2$, and $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{Q}$. We see that spin response is (a) incoherent: no sharp peak in $\chi''(\mathbf{q}, \Omega)$ as a function of frequency Ω and (b) commensurate: $\chi''(\mathbf{q}, \Omega)$ is peaked at $\mathbf{q} = \mathbf{Q}$ for a fixed Ω . Both results are in agreement with the data.¹¹

In a d -wave superconducting state, the bosonic self-energy is modified by the opening of a superconducting gap. Assembling the contributions from normal and anomalous bubbles, we obtain

$$\Pi(\mathbf{q}, \Omega) = \frac{i \gamma_\mathbf{q}}{2} \int \left(1 - \frac{\omega(\omega + \Omega) - \Delta_\mathbf{q}^2}{\sqrt{[\omega^2 - \Delta_\mathbf{q}^2][(\omega + \Omega)^2 - \Delta_\mathbf{q}^2]}} \right) d\omega. \quad (4)$$

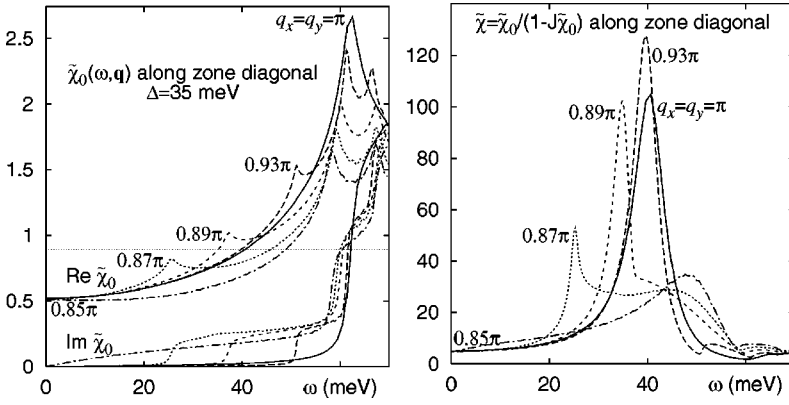
Here $\Delta_\mathbf{q}$ is the fermion gap at one of the two hot spots connected by \mathbf{q} , i.e., $\Delta_\mathbf{q} = \Delta(k = \mathbf{k}_{\text{hs}}(\mathbf{q}))$. By virtue of the $d_{x^2-y^2}$ symmetry, $\Delta(\mathbf{k}_{\text{hs}}(\mathbf{q})) = -\Delta(\mathbf{k}_{\text{hs}}(\mathbf{q}) + \mathbf{q})$, a condition we used in deriving Eq. (4).

In the presence of a superconducting gap with the d -wave symmetry, $\operatorname{Im} \Pi(\mathbf{q}, \Omega)$ vanishes discontinuously for $\Omega < 2|\Delta_\mathbf{q}|$.^{13,15,17-19} By virtue of the Kramers-Kronig relation, this discontinuity generates a nonzero $\operatorname{Re} \Pi(\mathbf{q}, \Omega)$ that is quadratic in Ω at low frequencies: $\Pi(\mathbf{q}, \Omega) \sim \gamma_\mathbf{q} \Omega^2 / \Delta_\mathbf{q}$. It is also essential that for any \mathbf{q} , $\Pi(\mathbf{q}, 0) = 0$: the opening of the d -wave gap does not change the magnetic correlation length. This result is not entirely surprising: spin-mediated d -wave pairing involves only fermions from opposite sublattices and thus does not affect the correlation of spin within the same sublattice.

Substituting $\Pi(\mathbf{q}, \Omega)$ into Eq. (1) we obtain

$$\chi(\mathbf{q}, \Omega) \propto c_\mathbf{q}^2 / [c_\mathbf{q}^2 (\xi^{-2} + \tilde{q}^2) - \Omega^2], \quad c_\mathbf{q}^2 = \Delta_\mathbf{q} / \gamma_\mathbf{q} \xi^2. \quad (5)$$

We see that in a d -wave superconductor the low-energy spin excitations are propagating, gapped magnonlike modes with the dispersion



$$\Omega_{\mathbf{q}}^2 = c_{\mathbf{q}}^2 (\xi^{-2} + |\mathbf{q} - \mathbf{Q}|^2). \quad (6)$$

Two comments are in order before we analyze this dispersion. First, Eqs. (5)–(6) are meaningful only if $c_{\mathbf{q}}\xi^{-1} < |\Delta_{\mathbf{q}}|$. Otherwise the use of a quadratic form for $\Pi(\mathbf{q}, \Omega)$ is not justified. Strictly speaking, near $\mathbf{q} = \mathbf{Q}$, $c_{\mathbf{q}}\xi^{-1} < \Delta_{\mathbf{q}}$ only at sufficiently strong coupling, when fermion self-energy cannot be neglected. On the other hand, a quadratic frequency dependence of $\text{Re} \Pi(\mathbf{q}, \Omega)$ at low Ω is merely a consequence of a vanishing $\text{Im} \Pi(\mathbf{q}, \Omega)$ for $\Omega < 2|\Delta_{\mathbf{q}}|$, where at strong coupling $\Delta_{\mathbf{q}}$ should be understood as the measured gap (i.e., a frequency where the spectral function has a δ -functional peak). We have explicitly verified that inclusion of strong coupling corrections into Eq. (4) only changes the overall factor in the resonance frequency.

Second, to verify that our analytical approach based on a linearized fermion dispersion around $\mathbf{k}_{\text{hs}}(\mathbf{q})$ captures the essential features of the spin susceptibility, we present in Fig. 2 numerical results for the bare particle-hole susceptibility $\tilde{\chi}_0(\mathbf{q}, \Omega)$ and its RPA counterpart $\tilde{\chi}(\mathbf{q}, \Omega) = \tilde{\chi}_0(\mathbf{q}, \Omega) / [1 - J\tilde{\chi}_0(\mathbf{q}, \Omega)]$. Both are calculated for a $d_{x^2-y^2}$ BCS superconductor with a tight-binding dispersion $\epsilon_{\mathbf{k}}$ inferred from ARPES data.¹⁸ We expect that, at low enough Ω , $\Pi(\mathbf{q}, \Omega)$ matches $\tilde{\chi}_0(\mathbf{q}, \Omega)$, modulo a regular function of q . Similarly, $\chi(\mathbf{q}, \Omega)$ should agree, at low frequencies, with $\tilde{\chi}(\mathbf{q}, \Omega)$. We see in Fig. 2 that $\text{Im} \tilde{\chi}_0(\mathbf{q}, \Omega) = 0$ for $\Omega < 2|\Delta_{\mathbf{q}}|$ and has a finite jump at $2\Delta_{\mathbf{q}}$, while $\text{Re} \tilde{\chi}_0(\mathbf{q}, \Omega)$ is quadratic in Ω at low frequencies, in agreement with what we obtained for $\Pi(\mathbf{q}, \Omega)$. The peaks in $\text{Re} \tilde{\chi}_0(\mathbf{q}, \Omega)$ at $2\Delta_{\mathbf{q}}$ are logarithmic singularities associated with the discontinuity of $\text{Im} \tilde{\chi}_0(2\Delta_{\mathbf{q}})$.^{13,15,17–19} They are softened because we used complex frequencies with a small imaginary part. These singularities are indeed present in $\Pi(\mathbf{q}, \Omega)$. A strong peak in the RPA susceptibility $\tilde{\chi}(\mathbf{q}, \Omega)$ is the resonance located at $1 = J \text{Re} \tilde{\chi}_0(\mathbf{q}, \Omega)$. This expression corresponds to $\Pi(\mathbf{q}, \Omega) = \xi^{-2} + (\mathbf{q} - \mathbf{Q})^2$ in our analytical approach. We have also verified that the static susceptibility $\tilde{\chi}_0(\mathbf{q}, 0)$ does not change between normal and superconducting states (i.e., ξ is not renormalized). High frequency features in Fig. 2 are due to the reduction in the number of scattering channels at the lowest frequencies at $q \neq Q$. For $q_x = q_y$, this reduction splits the 2Δ singularity at $q = Q$ into three singularities one of

FIG. 2. Left: real and imaginary parts of the numerically evaluated particle-hole bubble in a d -wave superconductor, $\tilde{\chi}_0(\mathbf{q}, \omega)$, at various $\mathbf{q} = (q, q)$, using a tight-binding fermion dispersion inferred from ARPES. The low-frequency features of $\tilde{\chi}_0(\mathbf{q}, \omega)$ are in agreement with our analytical results. Right: imaginary part of the corresponding full dynamical susceptibility, $\text{Im} \chi(\Omega, \mathbf{q})$. Observe that the resonance moves to lower frequencies when $\mathbf{q} \neq \mathbf{Q} = (\pi, \pi)$.

which is a true low-energy feature (which we considered analytically), and the other two rapidly move to high energies and become sensitive to the details of the fermion dispersion $\epsilon_{\mathbf{k}}$ far from the Fermi surface.

We now analyze Eq. (6). If $\Delta_{\mathbf{q}}$ and $\gamma_{\mathbf{q}}$ were independent of momentum, $c_{\mathbf{q}}$ would be constant and $\Omega_{\mathbf{q}}$ would be a conventional magnonlike dispersion. In this situation, the spin resonance would remain commensurate and exist only at $\Omega > \Omega_{\mathbf{Q}} = c_{\mathbf{Q}}/\xi$. However, in a d -wave superconductor, $\Delta_{\mathbf{q}}$ decreases and $\gamma_{\mathbf{q}}$ increases when \mathbf{q} deviates from \mathbf{Q} . This effect accounts for the downward shift of $\Omega_{\mathbf{q}}$. Furthermore, as $c_{\mathbf{q}}$ vanishes at $\mathbf{q} \rightarrow \mathbf{q}_0$, the resonance frequency $\Omega_{\mathbf{q}} \rightarrow 0$ regardless of the spin correlation length ξ . Quite generally, near \mathbf{q}_0 , $\text{Im} \chi(\mathbf{q}, \Omega)$ should have a peak at a frequency $\Omega_{\mathbf{q}} < \Omega_{\mathbf{Q}}$.

This dispersion near $\mathbf{q} = \mathbf{Q}$ is more complicated and depends on the values of ξ and \mathbf{q}_0 . To obtain a qualitative picture, we exploit the fact that $c_{\mathbf{q}}$ vanishes at $\mathbf{q} = \mathbf{q}_0$ and approximate the momentum dependence of $c_{\mathbf{q}}$ as $c_{\mathbf{q}}^2 \approx 1 - (\tilde{q}/\tilde{q}_0)^2$ where $\tilde{q} = |\mathbf{q} - \mathbf{Q}|$. Substituting this form into Eq. (6), we find

$$\Omega_{\mathbf{q}}/\Omega_{\mathbf{Q}} = 1 - (\tilde{q}/\tilde{q}_0)^2 [1 - (\tilde{q}_0\xi)^2] - (\tilde{q}/\tilde{q}_0)^4 (\tilde{q}_0\xi)^2. \quad (7)$$

We see that when $\tilde{q}_0\xi$ is small, the dispersion is negative for all momenta. When $\tilde{q}_0\xi$ is large, it first goes up, and then drops. In both cases, the residue of the peak in Eq. (5) scales as $c_{\mathbf{q}}$, i.e., it decreases and eventually vanishes as \mathbf{q} approaches \mathbf{q}_0 .

In Fig. 3 we plot $\Omega_{\mathbf{q}}$ (7) and the intensity of the peak for $\tilde{q}_0\xi = 2$. This value is consistent with optimally doped Bi2212 where $|\mathbf{q}_0| \approx 0.3\pi/a$ (Ref. 21), while $\xi = a - 2a$.²² $\Omega_{\mathbf{q}}$ in Eq. (7) is rather flat near \mathbf{Q} and rapidly drops away from \mathbf{Q} . The residue of the peak at the downturn is already much smaller than its value at $q \approx \mathbf{Q}$. Both features are consistent with the data.⁹

For comparison, we also present in Fig. 3 the dispersion and intensity of the collective mode obtained in the RPA calculation, Fig. 2. It exhibits qualitatively similar behavior which we interpret as evidence that the dispersion of the peak is insensitive to the details of the fermion dispersion at high energies. Note also that our estimate of the wave vector at which the energy and intensity of the neutron peak vanishes, $\mathbf{q}_0 \approx (0.8\pi, 0.8\pi)$, is roughly consistent with the cor-

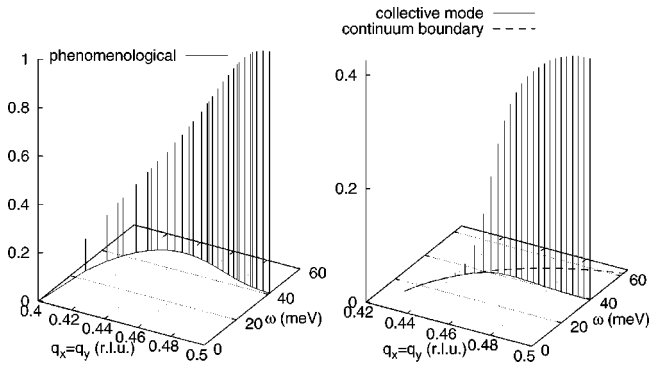


FIG. 3. Location and intensity of the resonance peak obtained from Fig. 2 (left) and from Eqs. (5) and (7) (right).

responding value found in neutron scattering, $\mathbf{q} = (\pi, 0.8\pi)$. Recall that, to simplify the analysis, we have only considered momenta along the zone diagonal.

Finally, we argue that with underdoping, $\tilde{q}_0\xi$ increases such that eventually $\Omega_{\mathbf{q}}$ should first increase with \tilde{q} and then drop very near $\mathbf{q} = \mathbf{q}_0$. This simply implies that at small \tilde{q} , the resonance continuously evolves into the magnon of a disordered antiferromagnet. Still, however, the drop near \mathbf{q}_0 should persist in the pseudogap phase and only disappear when the system develops a true long-range antiferromagnetic order. It would be of interest to verify experimentally whether such evolution takes place.

To summarize, we have demonstrated that the experimentally observed downturn of the resonant frequency⁹ away from (π, π) , accompanied by a rapid decrease of the peak intensity, occurs rather naturally when the resonance peak is

interpreted as a collective spin excitation in a d -wave superconductor. The unusual dispersion of the peak is related to a variation of the superconducting gap along the Fermi surface. We have argued that the resonant frequency vanishes at a certain $\mathbf{q} = \mathbf{q}_0$ that connects nodal points of the Fermi surface. Close to the commensurate wave vector $\mathbf{Q} = (\pi, \pi)$, the dispersion depends on the spin correlation length: If the latter is small, $|\mathbf{q}_0 - \mathbf{Q}|\xi \ll 1$, the resonance frequency $\Omega_{\mathbf{q}}$ decreases with $|\mathbf{q} - \mathbf{Q}|$. Conversely, when $|\mathbf{q}_0 - \mathbf{Q}|\xi \gg 1$, $\Omega_{\mathbf{q}}$ increases away from \mathbf{Q} , reminiscent of a magnon dispersion in a disordered antiferromagnet.

As we already said, the negative dispersion of the resonance peak has been earlier detected numerically in RPA studies for Hubbard and $t-J$ models.^{16–19} References 18 and 19 interpreted this dispersion as a consequence of the d -wave symmetry of the gap. Our analytical results are complementary to Refs. 18 and 19. In Ref. 17, however, the negative dispersion was argued to be the consequence of a closeness to a Van-Hove singularity (similar scenario was also considered in Ref. 23). We believe that the observed rapid drop of intensity away from (π, π) and the fact that the measured resonance frequency disappears at a momentum close to the one which connects nodal points at the Fermi surface argue in favor of a “ d -wave” explanation.

We acknowledge useful conversations with Ar. Abanov, J. C. Campuzano, P. Dai, M. R. Norman, and S. Sachdev. The research was supported in part by the NSF Grant No. DMR-9979749 and the US DOE Grant Nos. W-31-109-ENG-38 and DE-FG02-90ER40542. A.Ch. and O.T. thank the Materials Science Division of Argonne National Laboratory for hospitality during their stay at Argonne.

¹T. E. Mason, in *Handbook on the Physics and Chemistry of Rare Earths*, edited by K. A. Gschneider, Jr. *et al.*; cond-mat/9812297 (unpublished).
²J. C. Campuzano *et al.*, Phys. Rev. B **53**, R14 737 (1996).
³See, e.g., Physica C **341-348** (2000).
⁴J. Rossat-Mignod *et al.*, Physica C **185**, 86 (1991); H. F. Fong *et al.*, Phys. Rev. Lett. **75**, 316 (1995); P. Dai *et al.*, Science **284**, 1344 (1999).
⁵H. F. Fong *et al.*, Nature (London) **398**, 588 (1990).
⁶H. A. Mook *et al.*, Phys. Rev. Lett. **70**, 3490 (1993).
⁷S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, 1999).
⁸M. Arai *et al.*, Phys. Rev. Lett. **83**, 608 (1999).
⁹P. Bourges *et al.*, Science **288**, 1234 (2000).
¹⁰M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15 152 (2000).
¹¹See, for example, G. Aeppli *et al.*, Science **278**, 1432 (1997), and references therein.
¹²See, for example, M. R. Norman *et al.*, Phys. Rev. Lett. **79**, 3506 (1997); Z. X. Shen *et al.*, Science **280**, 259 (1998); P. V. Bogdanov *et al.*, *ibid.* **84**, 2581 (2000); H. M. Fretwell *et al.*, Phys. Rev. Lett. **84**, 4449 (2000).

¹³G. Stemmam *et al.*, Phys. Rev. B **50**, 4075 (1994); D. Z. Liu, Y. Zha, and K. Levin, Phys. Rev. Lett. **75**, 4130 (1995); I. Mazin and V. Yakovenko, *ibid.* **75**, 4134 (1995); A. J. Millis and H. Monien, Phys. Rev. B **54**, 16 172 (1996); N. Bulut and D. J. Scalapino, *ibid.* **53**, 5149 (1996); D. K. Morr and D. Pines, Phys. Rev. Lett. **81**, 1086 (1998); S. Sachdev and M. Vojta, Physica B **280**, 333 (2000); O. Tchernyshyov *et al.*, Phys. Rev. B **63**, 144507 (2001).
¹⁴J. Brinckmann and P. A. Lee, Phys. Rev. Lett. **82**, 2915 (1999).
¹⁵Ar. Abanov and A. Chubukov, Phys. Rev. Lett. **83**, 1652 (1999).
¹⁶D. Van der Marel, Phys. Rev. B **51**, 1147 (1995).
¹⁷F. Onufrieva and P. Pfeuty, cond-mat/9903097 (unpublished).
¹⁸M. R. Norman, Phys. Rev. B **61**, 14 751 (2000); *ibid.* **63**, 092509 (2001).
¹⁹Y.-J. Kao *et al.*, Phys. Rev. B **61**, R11 898 (2000).
²⁰Ar. Abanov *et al.*, cond-mat/0010403 (unpublished).
²¹T. Valla *et al.*, Science **285**, 2210 (1999).
²²See, e.g., D. J. Scalapino, Phys. Rep. **250**, 329 (1995); D. Pines, Z. Phys. B: Condens. Matter **103**, 129 (1997), and references therein.
²³A. A. Abrikosov, Phys. Rev. B **62**, 15 156 (2000).