

## Crossover from O(3) to O(4) behavior in weakly frustrated antiferromagnets

Andrey V. Chubukov

*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706  
and P.L. Kapitza Institute for Physical Problems, Moscow, Russia*

Oleg A. Starykh\*

*University of California at Davis, Davis, California 95616*

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We consider an anisotropic version of the  $CP^1$  model which describes frustrated quantum antiferromagnets with incommensurate spin correlations. We extend the two-component spinon field, describing lattice spins, to the  $M$ -component complex vector, and show, in the  $1/M$  expansion, that for arbitrary small incommensurability longitudinal and transverse stiffnesses tend to the same value as the system approaches the quantum critical point. For physical spins ( $M=2$ ), this yields O(4) critical behavior. However, if the spin structure is commensurate, the longitudinal stiffness is identically zero. In this case, the critical behavior is the same as in the O(3)  $\sigma$  model. We show how the critical exponents interpolate between O(3) and O(4) values near the transition. We also show that the competition between these two fixed points leads to a confinement-deconfinement transition at a finite temperature. [S0163-1829(96)50722-8]

In this paper, we address a specific issue concerning the nature of zero-temperature quantum phase transitions in quantum antiferromagnets: whether there is a qualitative change in the critical behavior when one adds a frustration to the Heisenberg antiferromagnet and makes spin configuration noncollinear. We argue that the critical behavior of collinear and noncollinear antiferromagnets is qualitatively different, even when local spin configuration differs from a Néel state by an arbitrary small amount. As a quantitative measure of this difference, we consider the ratio of the fully renormalized transverse and longitudinal stiffnesses,  $\rho_{\perp}/\rho_{\parallel}$ . In a collinear antiferromagnet, the longitudinal spin stiffness is identically zero because the Néel ordering is described by just one vector of antiferromagnetism,  $\vec{n}$ , and a rotation of  $\vec{n}$  around its equilibrium direction is *not* a symmetry transformation. On the other hand, in the ordered noncollinear state, one needs two unit vectors to describe the equilibrium spin configuration. In this case, a rotation of one vector around another is a legitimate symmetry transformation which is broken in the ordered state. As a result, there are two finite stiffnesses: one for two equivalent transverse spin-wave modes, and another for the longitudinal spin-wave mode. The action for noncollinear antiferromagnets can be written either in terms of the SO(3) rotation matrix, or in terms of two-component complex spinon field. Azaria *et al.* performed one-loop renormalization-group studies of the SO(3) action<sup>1</sup> and found that as the system approaches the critical point, the ratio of the fully renormalized stiffnesses tends to unity. Sachdev, Senthil, and one of us used the spinon description and have studied the critical behavior of frustrated antiferromagnets under the assumption that the bare transverse and longitudinal stiffnesses,  $\rho_{\perp}^0$  and  $\rho_{\parallel}^0$ , are close to each other.<sup>2</sup> They also found that as the system approaches the critical point, the ratio of the fully renormalized stiffnesses tends to unity. At criticality,  $\rho_{\perp} = \rho_{\parallel}$ , and the symmetry of the underlying action is enlarged from SU(2)  $\times$  U(1) to O(4) (see below). In this paper, we extend the

spinon approach to an arbitrary ratio of the bare stiffnesses. We will show that the O(4) critical behavior holds for arbitrary  $\rho_{\parallel}^0/\rho_{\perp}^0 \leq 1$ . We cannot say at the moment about how far the O(4) behavior extends to a region where  $\rho_{\parallel}^0 \gg \rho_{\perp}^0$ .

Our point of departure is the partition function for a frustrated antiferromagnet written in terms of spinon fields,  $\mathcal{Z} = \int Dz^* Dz \exp[-S]$ , where

$$S = 2\rho_{\perp}^0 \int d\tau d^2r \left[ |\partial_{\mu} z|^2 - \frac{\gamma}{4} (z^* \partial_{\mu} z - z \partial_{\mu} z^*)^2 \right]. \quad (1)$$

Here  $z$  is a two-component complex spinor field subject to constraint  $z_{\alpha}^* z_{\alpha} = 1$ ,  $\mu = \tau, x, y$ , and  $\gamma = (\rho_{\parallel}^0 - \rho_{\perp}^0)/\rho_{\perp}^0$ . For simplicity we choose units where  $\hbar = 1$  and set both spin-wave velocities to unity.

This effective action can be explicitly derived from semiclassical microscopic considerations<sup>3,4</sup> and the general macroscopic approach of Ref. 5. Note however that  $z$  quanta are *not* Schwinger bosons. The relation between  $z$  and the underlying spins is more complex and involves incommensurate ordering momentum.<sup>4,2</sup> The action in (1) is invariant under global SU(2) spin rotation, and is also invariant under certain types of lattice transformations. As shown in Ref. 2, this lattice symmetry is in essence identical to a lattice U(1) symmetry. We will thus refer to the total global symmetry of the effective action in (1) as SU(2)  $\times$  U(1). It is essential, however, that the U(1) gauge symmetry  $z(\vec{r}, \tau) \rightarrow z(\vec{r}, \tau) e^{i\phi(\vec{r}, \tau)}$  is broken provided  $\rho_{\parallel} \neq 0$ , i.e.,  $\gamma > -1$ . It is only present at  $\gamma = -1$  in which case the action in (1) describes collinear antiferromagnets. In this latter case, the description in terms of  $z$  quanta is equivalent to the description in terms of Schwinger bosons: the same action as in (1) is obtained if one introduces the U(1) gauge invariant Schwinger boson decomposition  $n^a = b_{\alpha}^{\dagger} \sigma_{\alpha\beta}^a b_{\beta}$  into the par-

tion function of the  $O(3)$   $\vec{n}$ -field model.<sup>2,6</sup>  $\gamma \ll 1$  limit of the action (1) was studied in detail in Ref. 2. Here we focus on a region near  $\gamma = -1$ .

To perform  $1/M$  expansion, we need to generalize the action to large  $M$ . We generalize the doublet  $z$  to the  $M$ -component complex vector, rescale the  $z$  field to  $z \rightarrow z/\sqrt{M}$  (such that  $z_\alpha^* z_\alpha = 1$ ,  $\alpha = 1, 2, \dots, M$ ), and introduce the coupling constant  $g = M/2\rho_\perp^0$ . We further introduce the Hubbard-Stratonovich vector gauge field  $A_\mu$  to decouple the quartic term, and introduce a constraint into the action using the integral representation of the  $\delta$  function. We then obtain

$$S = \frac{1}{g} \int d^2r \int_0^{1/T} d\tau \mathcal{L}, \quad (2)$$

$$\mathcal{L} = \left[ (\partial_\mu \chi)^2 + (\partial_\mu \eta)^2 + \sum_{\alpha=1}^{2M-2} (\partial_\mu \phi_\alpha)^2 - 2\sqrt{M} \sigma_0 \eta \partial_\mu A_\mu - 2A_\mu (\chi \partial_\mu \eta - \eta \partial_\mu \chi + \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 + \dots) + A_\mu^2 M(1+r) + i\lambda \left( 2\sqrt{M} \sigma_0 \chi + \chi^2 + \eta^2 + \sum_{\alpha=1}^{2M-2} \phi_\alpha^2 \right) \right]. \quad (3)$$

This decomposition of the  $z$  field implies that the variables  $\phi_\alpha$  describe  $2M-2$  transverse fluctuations,  $\eta$  is a variable for a longitudinal mode, and  $\chi$  describes fluctuations in the direction of the condensate.

Our first goal is to integrate out fluctuations of  $\chi$ ,  $\eta$ , and  $\phi_\alpha$  and to obtain the effective action for collective variables  $\mathbf{A}$  and  $\lambda$ . The integration over the longitudinal and transverse fluctuations yields contributions to the effective action which are linear in  $M$ , whereas the  $\chi$  field contributes only a sub-leading,  $O(1)$ , term which can be safely neglected in the leading order calculations in  $1/M$ . Performing the Gaussian integration over fluctuating fields and using the constraint equation at  $M = \infty$ , we obtain the effective action for the gauge fluctuations in the form  $S_A = (M/2) \int d^3 \mathbf{q} \Pi_{\mu\nu}(q) A_\mu(q) A_\nu(-q)$ , where

$$\Pi_{\mu\nu}(q) = 2 \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2} - \int \frac{d^3 k}{(2\pi)^3} \frac{(2k_\mu + q_\mu)(2k_\nu + q_\nu)}{k^2(k+q)^2} + 2\delta_{\mu\nu} \frac{2(r+\sigma_0^2)}{g} - \frac{2\sigma_0^2}{g} \frac{q_\mu q_\nu}{q^2}. \quad (4)$$

The first two terms in (4) are the components of the polarization operator of the  $O(3)$   $\sigma$  model,  $\Pi_{\mu\nu}^\phi(q)$ , which should be massless due to the gauge invariance of the latter. Using the Pauli-Willars regularization, we obtain  $\Pi_{\mu\nu}^\phi(q) = (\delta_{\mu\nu} - q_\mu q_\nu / q^2) q / 16$ . Collecting all terms in (4) and inverting the result we find for the gauge field propagator

$$\mathcal{L} = \left[ \left( \frac{1}{1+r} \right) |(\partial_\mu - iA_\mu)z|^2 + \frac{r}{1+r} |\partial_\mu z|^2 + i\lambda(|z|^2 - M) \right],$$

where we introduced  $r = -(1+\gamma)/\gamma$ .

The most straightforward way to compute the ratio of the fully renormalized stiffnesses, which we will follow, is to perform calculations in the ordered state at  $T=0$ . This state is realized for  $g$  smaller than the critical coupling  $g_c$ . Assume that the first component of  $z$  is condensed. We then write  $z = (\bar{\sigma}, \pi_\alpha)$ , and represent  $\bar{\sigma}$  as a sum of the condensed part,  $\sqrt{M}\sigma_0$ , and fluctuation  $\sigma$  around it,  $\bar{\sigma} = \sqrt{M}\sigma_0 + \sigma$ . It is also convenient to introduce pairs of real variables instead of complex variables  $\sigma$  and  $\pi_\alpha$ :  $\sigma = \chi + i\eta$ ,  $\pi_\alpha = \phi_{2\alpha-1} + i\phi_{2\alpha}$ , and rescale gauge field as  $A_\mu \rightarrow (1+r)A_\mu$ . Substituting these expressions into the action we find

$$D_{\mu\nu}(q) = \frac{1}{M} \left( \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left[ \frac{q}{16} + \frac{2(r+\sigma_0^2)}{g} \right]^{-1} + \frac{q_\mu q_\nu}{q^2} \frac{g}{2r} \right). \quad (5)$$

Notice that the longitudinal part of the propagator appears only due to incommensurability.<sup>9</sup> This is a direct consequence of the fact that incommensurability breaks the gauge symmetry of the action (3).

The propagator of the constraint field,  $\Pi^{-1}(q)$ , can be calculated in a similar way. We found that, to leading order in  $1/M$ ,  $\Pi(q)$  is independent on  $r$  and has the same form as in Ref. 2:  $\Pi^{-1}(q) = 8q^2/(q+16\sigma_0^2/g)$ .

We proceed now with the calculations of the stiffnesses at  $M = \infty$ . The two stiffnesses can be extracted from the long-distance behavior of the propagators of the transverse and longitudinal fields:  $G_{\phi_\alpha}^{-1}(q) = \rho_\perp q^2$ ,  $G_\eta^{-1}(q) = \rho_\parallel q^2$ . The computation of these propagators is straightforward. Transverse fields  $\phi_\alpha$  do not directly couple to the condensate, and all corrections to the free-particle propagator have relative  $1/M$  smallness. Hence full  $M = \infty$  propagator coincides with the bare one  $G_{\phi_\alpha}(q) = g/(2q^2)$ , i.e.,  $\rho_\perp = 2/g$ . The longitudinal ( $\eta$ -field) propagator is, however, different from  $G_{\phi_\alpha}$  already at  $M = \infty$  because of the coupling term  $\propto \sqrt{M} \sigma_0 \eta \partial_\mu A_\mu$  in (3). This term leads to the finite  $M = \infty$  correction,

$$G_\eta(q) = \frac{g}{2q^2} + M \sigma_0^2 \frac{q_\mu q_\nu}{q^4} D_{\mu\nu}(q) = \frac{g}{2q^2} \frac{r+\sigma_0^2}{r}, \quad (6)$$

so that  $\rho_{\parallel} = 2r/g(r + \sigma_0^2)$ . We emphasize that Eq. (6) is an exact  $M = \infty$  result, not an expansion around the free-particle expression.

For the ratio of the fully renormalized stiffnesses we then obtain

$$\bar{\gamma} = \frac{\rho_{\parallel} - \rho_{\perp}}{\rho_{\perp}} = -\frac{\sigma_0^2}{r + \sigma_0^2} \equiv \gamma \frac{\sigma_0^2}{1 + \gamma - \gamma \sigma_0^2}. \quad (7)$$

This is the key result of the  $M = \infty$  consideration. We see that as long as the longitudinal stiffness is finite (i.e.,  $\gamma > -1$ ), the ratio of the fully renormalized stiffnesses approaches 0 as the system moves to the critical point,  $\sigma_0 \rightarrow 0$ . At criticality,  $\bar{\gamma} = 0$ , and the renormalized action (1) reduces to that for the  $O(2M)$   $\sigma$  model. At the same time, if  $\rho_{\parallel}^0 = 0$  (i.e.,  $\gamma = -1$ ), then  $\bar{\gamma} = \gamma = -1$ , and the renormalized action retains the symmetry of the isotropic  $CP^{M-1}$  model. Alternatively stated, for  $\rho_{\parallel} = 0$ , which is the case for a collinear antiferromagnet, the system is in the basin of attraction of the  $CP^{M-1}$  fixed point. However, an arbitrary small amount of noncollinearity drives the system to the  $O(2M)$  fixed point.

Our next goal is to compute  $1/M$  corrections to the ratio of stiffnesses. These corrections come from self-energy diagrams which involve exchange by fluctuations of both the constraint and the gauge field. The computational steps are rather involved, but conceptually are similar to those discussed in Refs. 7,2. For  $r \gg \sigma_0^2$  we obtained

$$G_{\phi}^{-1} = \frac{2q^2}{g} \left( 1 - \frac{4}{3\pi^2 M} L \right), \quad (8)$$

$$G_{\eta}^{-1} = \frac{2q^2}{g} \left( 1 - \frac{4}{3\pi^2 M} L \right) \left( 1 - \frac{\sigma_0^2}{r} \left( 1 + \frac{28}{3\pi^2 M} L \right) \right),$$

where  $L = \ln(g_c - g)/g_c$ , and  $\sigma_0$  is related to  $g_c - g$  as in the  $O(2M)$  model,<sup>7</sup>  $\sigma_0^2 = (1 - g/g_c)^{1-4/(\pi^2 M)}$ . This is merely a consequence of the fact that the propagator of the constraint field does not depend on  $r$ . For the ratio of the stiffnesses we then obtain  $\bar{\gamma} = (\gamma/(1 + \gamma))(1 - g/g_c)^{1+16/(3\pi^2 M)}$ . For  $\gamma \ll 1$ , this reproduces the result of Ref. 2. We see that the  $1/M$  corrections only speed up the flow to the  $O(2M)$  fixed point.<sup>11</sup> For  $r \leq \sigma_0^2$ , the expression for  $\bar{\gamma}$  is rather involved and we refrain from presenting it.

For completeness, we also computed critical exponents in the  $1/M$  expansion. For  $\gamma = -1$  (i.e.,  $r = 0$ ) spinon correlation function and order parameter possesses  $\eta = -20/\pi^2 M$ ,  $\nu = 1 + 16/\pi^2 M$ ,  $2\beta = 1 - 4/\pi^2 M$ . The actual spin susceptibility is a convolution of two spinon fields, and it has different critical exponents  $\bar{\eta}$ ,  $\bar{\nu}$ , etc. We computed the spin susceptibility to order  $1/M$  and found  $\bar{\eta} = 1 - 32/\pi^2 M$ ,  $\bar{\nu} = \nu$ ,  $\bar{\beta} = 1 + O(1/M^2)$ . The result for  $\bar{\eta}$  has been reported by us previously.<sup>6</sup>

For  $\gamma > -1$  (i.e.,  $r > 0$ ), the gauge field acquires a mass, and the self-energy terms associated with the exchange of gauge field fluctuations are no longer singular. We have checked that the spinon fields now possess  $O(2M)$  exponents<sup>7</sup>:  $\eta = 4/3\pi^2 M$ ,  $\nu = 1 - 16/3\pi^2 M$ ,  $2\beta = 1 - 4/\pi^2 M$ . For spin-spin correlation function and magnetization (at arbitrary  $\gamma$ ) we reproduced the results of Ref. 2:  $\bar{\beta} = 1 + O(1/M^2)$ ,  $\bar{\nu} = 1 - 16/3\pi^2 M$ ,  $\bar{\eta} = 1 + 32/3\pi^2 M$ .

So far we were discussing the zero-temperature critical properties of incommensurate antiferromagnets. As the exponent  $\bar{\eta}$  is finite for both  $\gamma = -1$  and  $\gamma > -1$ , the staggered static spin susceptibility,  $\chi(q)$ , at criticality possesses a branch-cut singularity independent on whether the system is in the basin of attraction of  $O(3)$  or  $O(4)$  fixed points. From this perspective, the difference between  $O(3)$  and  $O(4)$  critical behavior at  $T = 0$  is only a quantitative but not qualitative one. In general, however, the descriptions of the system in terms of  $\vec{n}$  field and in terms of spinons are fundamentally different: in the first case the excitations necessary possess integer spin while in the latter one can have excitations with either integer or half-integer spin depending on whether spinons are confined or not. We now show that the presence of two fixed points at  $T = 0$  gives rise to a confinement-deconfinement transition within the *disordered* region. Consider for definiteness the renormalized-classical (RC) regime,  $g < g_c$ .<sup>8</sup> Here the ground state is ordered, but an arbitrary small temperature leads to the restoration of the spin rotational symmetry which implies that one can no longer distinguish between transverse and longitudinal fluctuations. Consider first the collinear antiferromagnet,  $\gamma = -1$ . Then the critical behavior is governed by the  $O(3)$  exponents. As the isotropic  $CP^1$  model is isomorphic to the  $O(3)$   $\sigma$  model, one should obtain the same result within the  $\vec{n}$  field and the spinon description. We recently demonstrated<sup>6</sup> that this is indeed the case. Despite the fact that effective action (1) yields a branch cut behavior of  $\chi(q)$  at the mean-field ( $M \rightarrow \infty$ ) level, the gauge field fluctuations, which appear at the  $1/M$  level, confine spinons into pairs with integer spin. These bound states of spinons yield poles in  $\chi(q)$ , and the long-distance behavior of spin correlators totally consistent with the  $\vec{n}$ -field description. The branch-cut to pole transformation is a direct consequence of the gauge invariance of the action (1) at  $\gamma = -1$ . Unbroken gauge invariance leads to gapless gauge field fluctuations, which give rise to the unbounded long-range confining potential between spinons.

Let us consider now what happens at  $\gamma > -1$ . We have shown in Ref. 6 that the static staggered susceptibility  $\chi(q)$  is proportional to the Green's function  $\Psi(q, x=0)$  of the effective inhomogeneous Schrodinger equation

$$\left( -\frac{d^2}{dx^2} + V(x) + \delta^2 \right) \Psi(x) = \delta(x), \quad (9)$$

where  $\delta^2 = q^2/4 + m_0^2$ , and  $m_0$  is the gap in the spinon spectrum. At  $M = \infty$ , the confining potential  $V(x)$  vanishes, and  $\chi(q)$  has a branch-cut singularity at  $\delta = 0$ . At finite  $M$ ,  $V(x)$  is given by the regularized Fourier transform of the transverse part of the gauge field propagator<sup>6</sup>

$$V(x) = \frac{m_0 T}{M} \int_{-\infty}^{\infty} \frac{dk}{2\pi} D_{\text{trans}}(k) (1 - e^{-ikx}). \quad (10)$$

In the RC regime, we found that in the small momentum limit,  $D_{\text{trans}}(k)$  is

$$D_{\text{trans}}(k) = \frac{1}{M} \left[ \frac{k^2 T}{12\pi m_0^2} + \frac{2r}{g} \right]^{-1}. \quad (11)$$

The appearance of the mass term  $2r/g$  in  $D_{\text{trans}}(k)$  makes the would-be-confining potential between spinons short ranged:  $V(x) = (6\pi m_0^3/Mm_A)(1 - e^{-|x|m_A})$ , where  $m_A = m_0\sqrt{24\pi r/gT}$ . Our key observation is that the gauge field mass,  $m_A$ , is inversely proportional to the temperature. At high temperatures  $m_A$  is small, and the potential is linear in  $|x|$  up to large scales leading to a strong binding of spinons.<sup>6</sup> However, as  $T \rightarrow 0$ ,  $m_A \rightarrow \infty$ , and  $V(x)$  reduces to zero in which case no bound states exist. There exists therefore a critical value of the gauge field mass when the attraction between spinons becomes too weak to bound them into pairs. It is not difficult to show that all excited bound states disappear already at  $(6\pi/M)^{1/3}m_0/m_A \sim O(1)$ . However, the first bound state survives as long as  $V(x \rightarrow \infty)$  remains positive. Careful analysis of the homogeneous version of Eq. (9) with full  $D_{\text{trans}}(k)$ , performed by Campostrini and Rossi,<sup>9</sup> shows that the first bound state disappears when  $(2r/gT) = \frac{1}{\pi}$ . Confinement-deconfinement temperature is then  $T^* = 4\pi\rho_{\perp}^0 r/M$ .

Above  $T^*$  there is a confinement, and the staggered static spin susceptibility  $\chi(q)$  has a pole singularity,  $\chi_{\text{conf}}(q) = A/(q^2 + m^2)$  ( $A \rightarrow 0$  as  $T \rightarrow T^*$ ), which translates into  $\chi(r) \propto r^{-1/2}e^{-rm}$  at large distances. Below  $T^*$ , spinons are deconfined, and  $\chi(q)$  has only a branch-cut singularity,  $\chi_{\text{deconf}}(q) \sim (q^2 + m^2)^{-1/2}$  which implies that at large distances  $\chi(r) \propto r^{-1}e^{-rm}$ . There is therefore a real change in the behavior of the physical observable at  $T^*$ ; however, we do not expect that there will be any changes in the thermodynamic quantities at this temperature.

We also caution that our low-energy analysis is restricted to the neighborhood of the  $CP^1$  fixed point, where the gauge field mass is small at  $T \sim T^*$ . The fate of the transition line at larger  $r$  is unknown simply because one cannot use the Hubbard-Stratonovich decoupling when the gauge field mass becomes comparable to the upper cutoff of the theory.<sup>12</sup>

The results of the present paper are in agreement with Ref. 2 and with the results of other authors. Transformation from free spinons at  $r \neq 0$  to the confined ones at  $r = 0$  was observed by Wiegmann<sup>10</sup> in the exact solution of the two-dimensional (2D) classical  $O(3)$  problem. Deconfinement of spinons in the  $T = 0$  quantum-disordered phase with incommensurate spin correlations, due to the appearance of the gauge field mass, was discussed by Sachdev and Read.<sup>13</sup> Classical  $d$ -dimensional versions of the actions (1) and (3)

were recently studied by Azaria *et al.*<sup>12</sup> by the renormalization group and  $1/M$  analysis in all dimensions between 2 and 4. One of their key findings is that all models with  $\gamma \neq -1$  are asymptotically equivalent at long distances to the  $O(2M)$  model. This is in complete agreement with our results. They also argued that though the low-energy physics is controlled by  $O(2M)$  fixed point, the high-energy (short distance) behavior for weak incommensurability is still controlled by the  $CP^1$  fixed point—this is also consistent with our result that confinement persists at high temperatures. They, however, did not explicitly discuss the confinement-deconfinement transition for the spin susceptibility.

To conclude, in this paper we analyzed the crossover from the  $CP^1 \equiv O(3)$  to the  $O(4)$  critical behavior in a model of weakly frustrated quantum antiferromagnet. We have shown, in the  $1/M$  expansion, that for an arbitrary small bare longitudinal stiffness, the system flows away from the  $CP^{M-1}$  fixed point towards the  $O(2M)$  fixed point. The same type of critical behavior was found previously for the case where the bare longitudinal and transverse stiffnesses were close to each other.<sup>2</sup> It is therefore likely that the  $O(2M)$  critical behavior [i.e., the  $O(4)$  behavior for the physical case of  $M = 2$ ] holds at least for all  $\rho_{\parallel}^0 \leq \rho_{\perp}^0$ . An unresolved issue is whether the  $O(4)$  behavior holds for an arbitrary *large* ratio of  $\rho_{\parallel}^0/\rho_{\perp}^0$ , or there is a crossover to a different kind of critical behavior with possible binding of spinons. This last issue is interesting on its own grounds but it is also possibly related to the controversy surrounding the critical behavior of stacked triangular antiferromagnets which, as numerical studies indicate, possess critical exponents different from the  $O(4)$  ones.<sup>14</sup> The ratio of the stiffnesses in 2D triangular antiferromagnets is not known exactly: noninteracting spin-wave calculations yield  $\rho_{\parallel}^0/\rho_{\perp}^0 = 2$  but first  $1/S$  corrections are not small and substantially reduce this ratio.<sup>15</sup>

We also have shown that the competition between the two zero-temperature fixed points leads to a confinement-deconfinement transition for static spin susceptibility at a finite temperature. Numerical studies of this phenomena are very desirable.

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\*On leave from the Institute for High Pressure Physics, 142092, Troitsk, Moscow Region, Russia.

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