Condensation energy and optical properties of the cuprates

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Outline

- Condensation energy in strongly coupled superconductors
  - where it comes from
  - the magnitude
- A relation between E_c and optical properties
  - partial sum rule for the relaxation rate $1/\tau(\omega)$
  - high frequency behavior of the conductivity $\sigma(\omega)$

Condensation energy

$E_c \sim 5 - 10K$, decreases with underdoping (Loram *et al*)

Questions:

- what is the origin of E_c?
- from what energies it comes from?
- why it is so low?

Suggestions:

- predominantly magnetic (resonance peak)
- predominantly electronic, via the gain in the kinetic energy

Spin-mediated $d-$wave pairing + Eliashberg formalism

electrons, their spin collective modes, spin-fermion coupling

Parameters: one dimensionless coupling λ and one overall scale $\bar{\omega}$.

- weak coupling $(\lambda \ll 1)$ $E_c = -N\frac{\Delta^2}{2}$ (BCS theory)
- strong coupling $(\lambda \geq 1)$ - the normal state is not a Fermi liquid at typical $\omega \sim \Delta$
  - feedback on electrons (they propagate more freely in the sc state than in the normal state)
  - spin collective modes change from diffusive to propagating

$$E_c = \Omega_{sc} - \Omega_n \;= \delta\Omega_{el} + \delta\Omega_{spin}$$

The electronic part

$$\delta\Omega_{el} = -N_f \pi T \sum_m \Big(\sqrt{\widetilde{\Sigma}^2_{s,\omega_m} + \Phi^2_{\omega_m}} - |\widetilde{\Sigma}_{n,\omega_m}| + |\omega_m| \frac{|\widetilde{\Sigma}_{s,\omega_m}| - \sqrt{\widetilde{\Sigma}^2_{s,\omega_m} + \Phi^2_{\omega_m}}}{\sqrt{\widetilde{\Sigma}^2_{s,\omega_m} + \Phi^2_{\omega_m}}} \Big)$$

The spin part

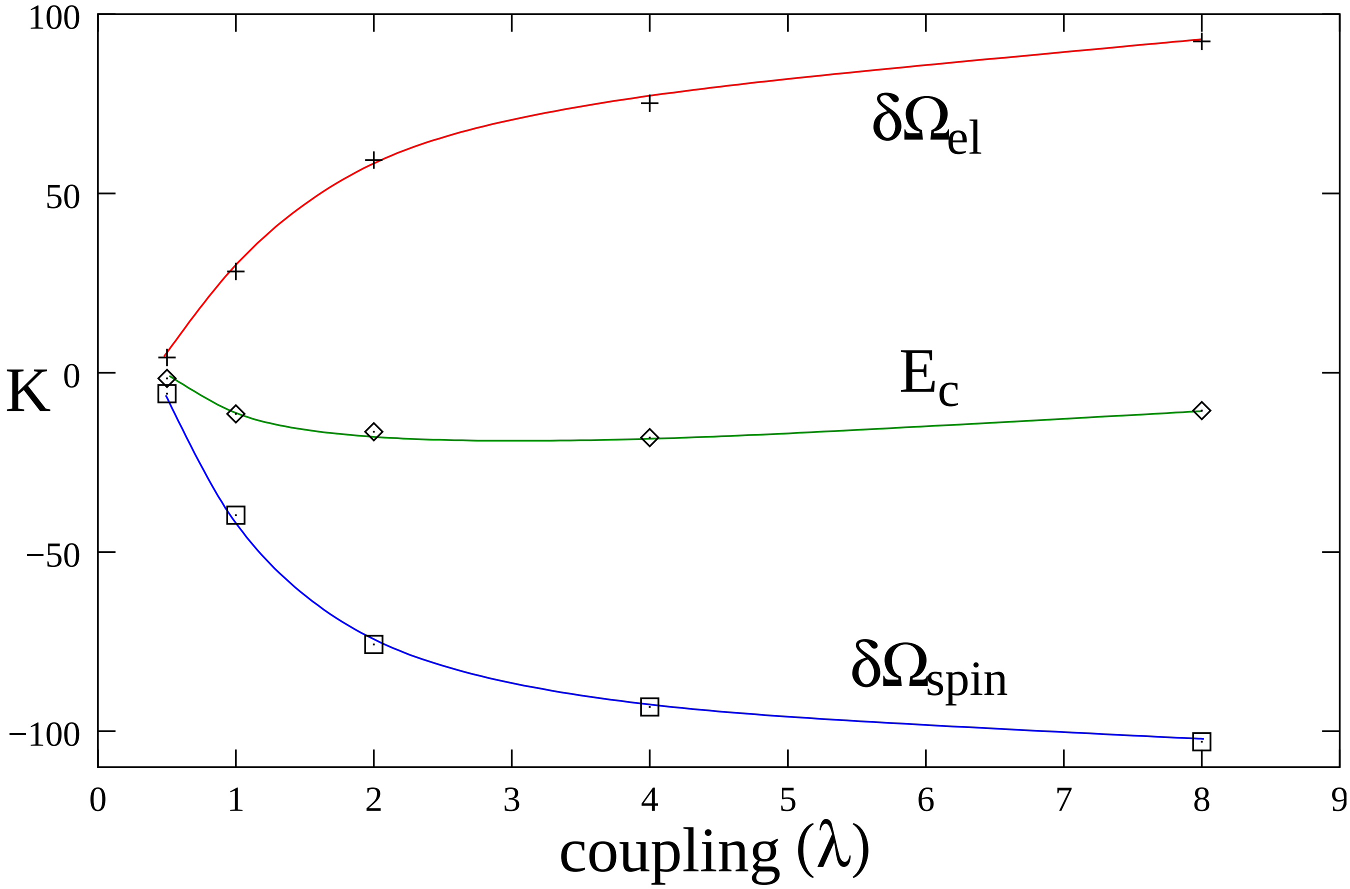
$$\delta\Omega_{spin} = -3N_s \pi T \sum_m \left(\Pi_{s,\omega_m} - \Pi_{n,\omega_m} + \frac{\bar{\omega}}{4\lambda^2} \log \frac{\bar{\omega} - 4\lambda^2 \Pi_{s,\omega_m}}{\bar{\omega} - 4\lambda^2 \Pi_{n,\omega_m}} \right)$$

$\widetilde{\Sigma} = \omega + \Sigma(\omega)$, $\Phi(\omega)$, and the spin polarization operator $\Pi(\omega)$ are mutually related!

Eliashberg, Luttinger and Ward (a general formula)

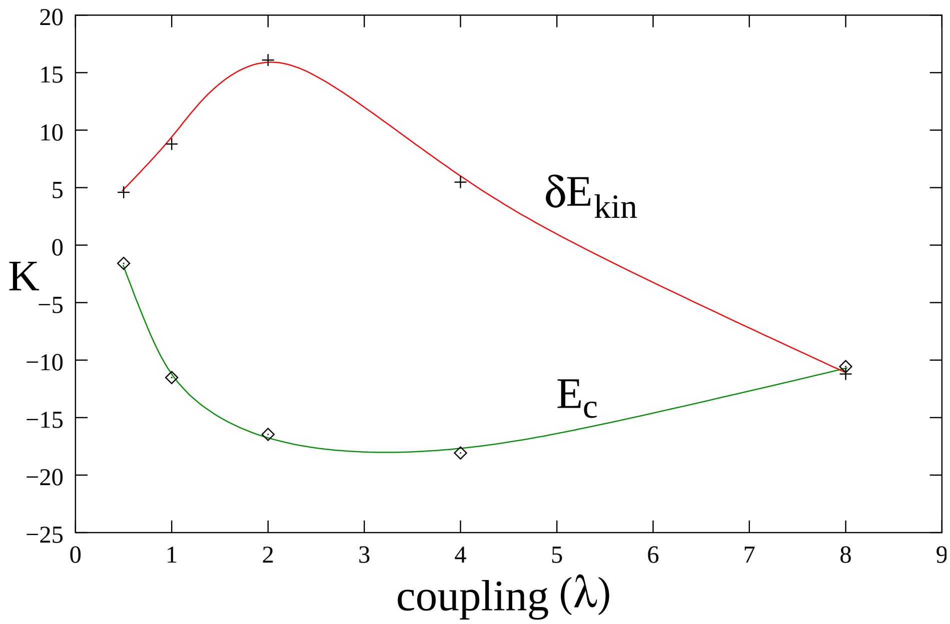
Wada, Bardeen and Stephen, Marsiglio and Carbotte (phonon case; negligible feedback on phonons)

Opt. doping: $\lambda \sim 1.5 - 2, \;\; \bar{\omega} \sim 200 meV$



- the spin part is negative (the effect of the resonance peak)
- the electronic part is positive (less magnetic scattering of electrons)
- a substantial cancellation between the two parts

Kinetic energy

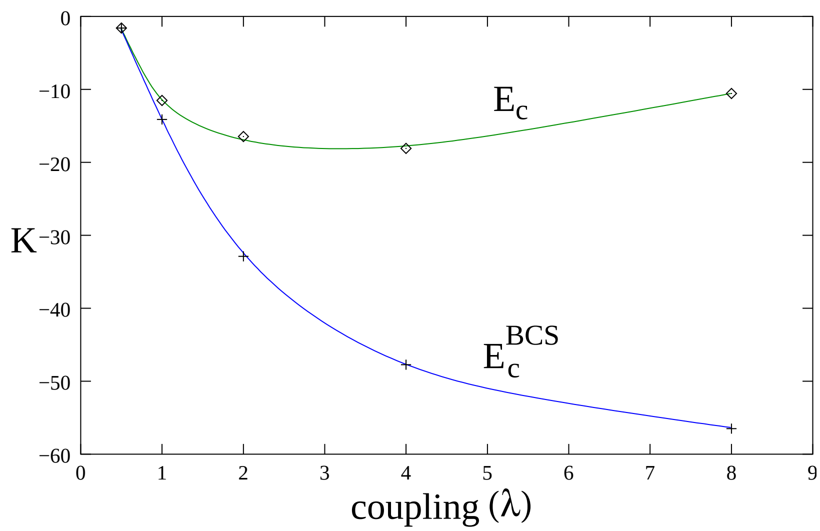


- the kinetic energy changes sing near optimal doping
- the total E_c can be viewed as due to the gain of E_{kin}
- it can be equally viewed as due to the gain in the spin potential energy

Essential: $E_c \sim 10K$, it comes from frequencies $O(\bar{\omega})$

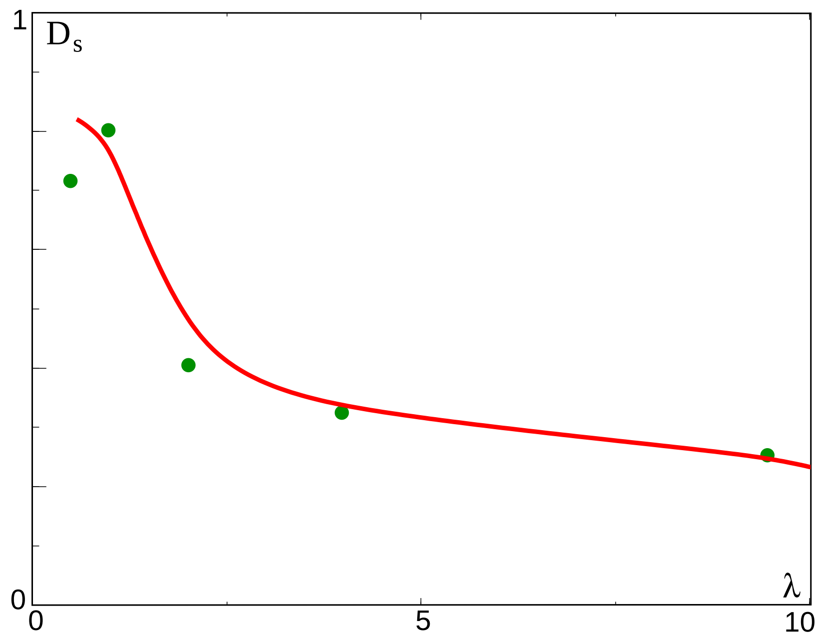
BCS theory: $E_c = -N_f \frac{\Delta^2}{2}$

$\Delta \sim 40meV \;\; N_f \sim 1st/eV \;\; \to E_c \sim 10K$



- E_c has a maximum at around optimal doping, and then decreases
- The reduction of E_c is due to the softening of the longitudinal gap fluctuations (infinite number of solutions for $\Delta(\omega)$ at $\lambda = \infty$)
- The reduction of E_c is a part of the pseudogap story

The decrease in E_c is correlated with the decrease of the superfluid stiffness



This correlation does not exist in a dirty superconductor – there the stiffness decreases with increasing impurity strength, but E_c remains exactly as in BCS theory

Differential sum rule for the relaxation rate $1/\tau(\omega)$

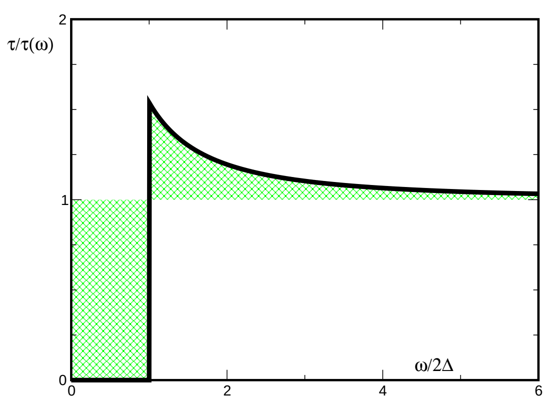
$$\frac{1}{\tau(\omega)} = \frac{4\pi}{\omega_{pl}^2} \mathrm{Re} \frac{1}{\sigma(\omega)}$$

By Kubo formula

$$\frac{1}{\tau(\omega)} = -\mathrm{Im} \frac{\omega}{\Pi(\omega)}$$

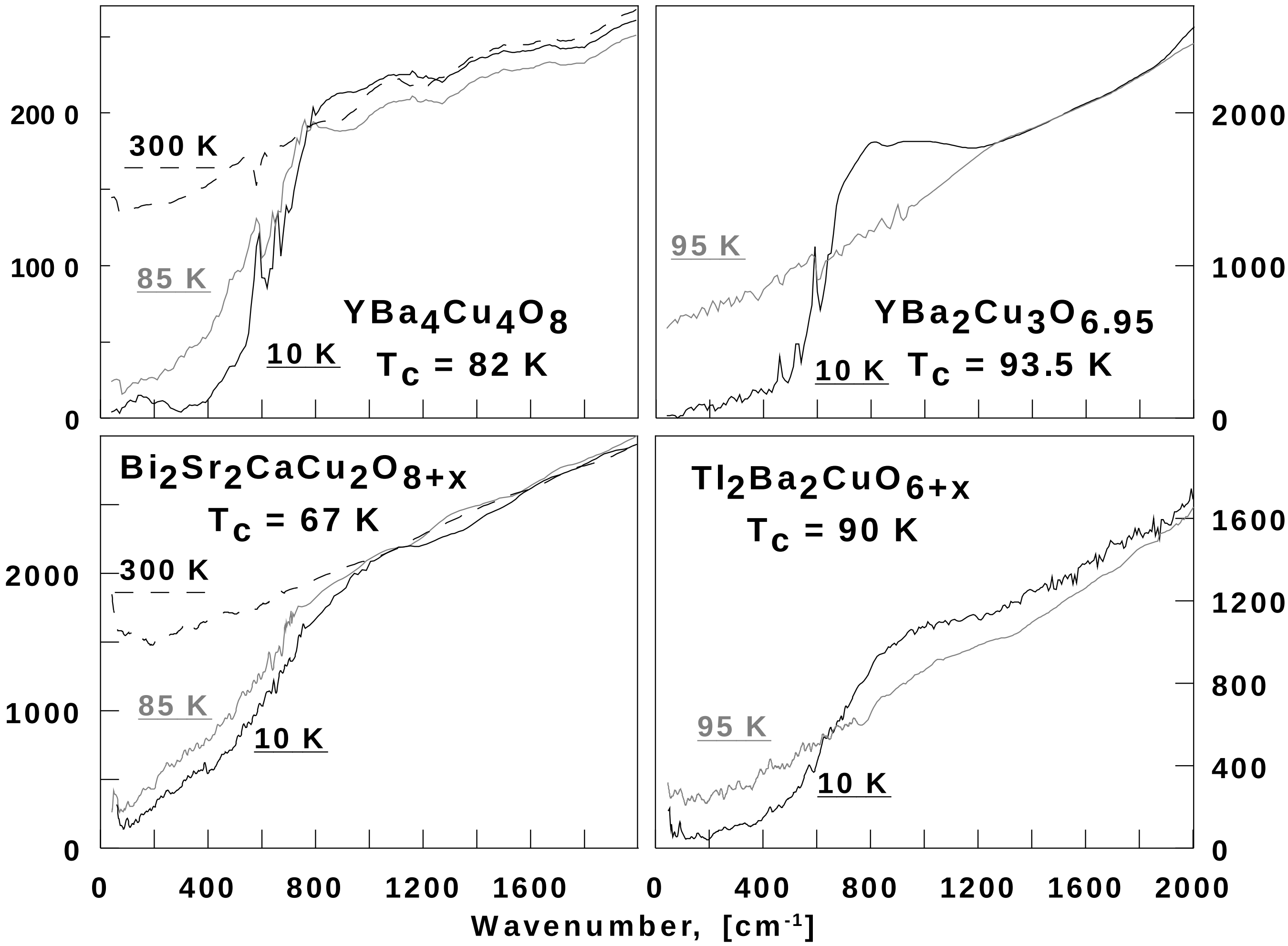
$$I_\tau = \int_0^\infty \left[\frac{1}{\tau(\omega, T_1)} - \frac{1}{\tau(\omega, T_2)} \right] d\omega = 0$$

In a dirty BCS superconductor

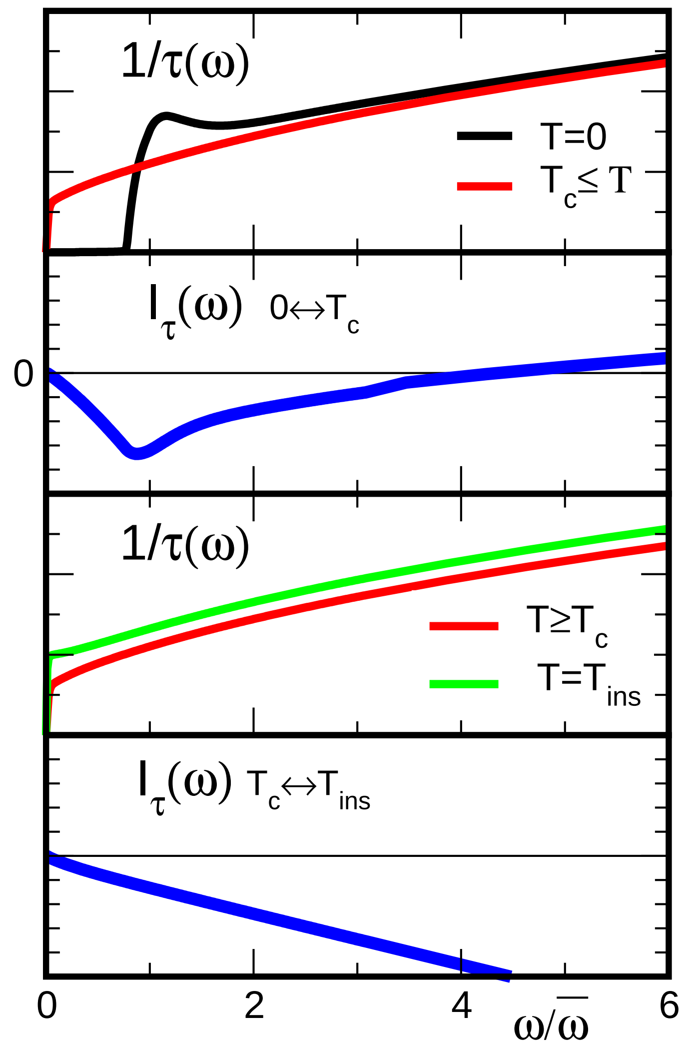


For any ratio γ/Δ, an overshoot occurs at $\omega \sim 2\Delta$.

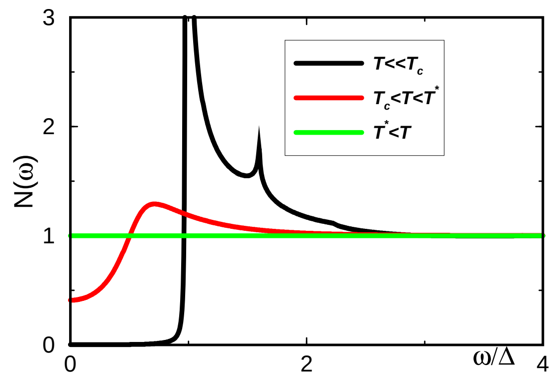
Experiment (Basov *et al*)



- $0 < T < T_c$ – differential sum rule is satisfied
- $T_c < T < T^*$ – sum rule is violated

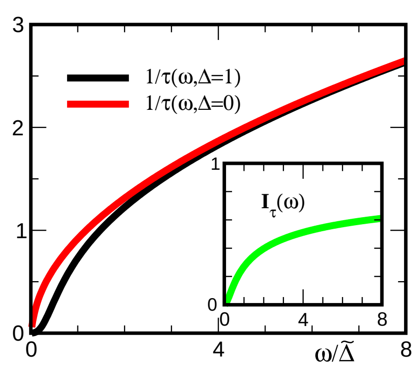


- $0 < T < T_c$ – differential sum rule is exhausted at about 10Δ
- $T_c < T < T^*$ – sum rule is violated up to the bandwidth.



- At $T_c < T < T^*$ almost no feedback on fermions (no such effect in a dirty superconductor).

A toy model: gap develops but fermions retain the normal state, NFL form $\Sigma \sim \sqrt{\omega}$



Same behavior as in the spin-fermion model above T_c.

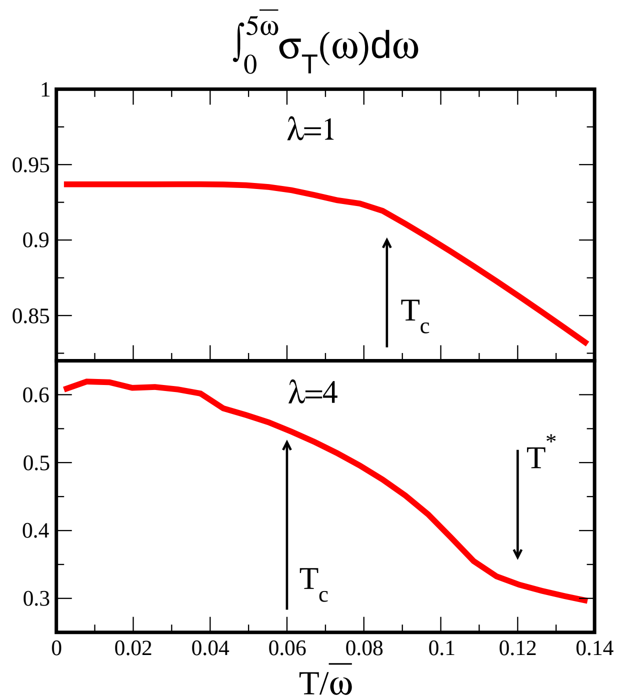
Optical conductivity at high frequencies

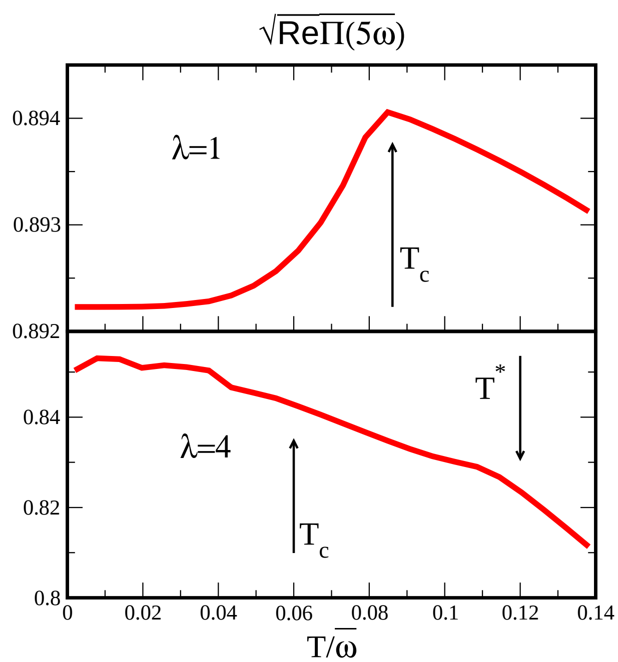
(D. van-der-Marel et al)

- enhancement of the spectral weight below $1eV$ (removal of the spectral weight between $1eV$ and $2ev$)
- enhancement of the plasma frequency (blue shift)

$$\epsilon(\omega) = 1 - \frac{4\pi ne^2/m}{\omega^2} \mathrm{Re}\Pi(\omega)$$
$$\omega_{pl}(T) = \frac{4\pi ne^2}{m} [\mathrm{Re}\Pi(\omega)]^{1/2}$$

These two effects are not present in a dirty superconductor





The transformation of the spectral weight involves energies above $1eV$.

Conclusions

The pseudogap physics is determined by scales $O(\bar{\omega})$ that can exceed the bandwidth. Superconductivity comes from much lower scales.

- the condensation energy $E_c \sim 10K$ comes from $O(\bar{\omega})$, and is the result the interplay between magnetic and electronic contributions
- the differential sum rule above T_c is not satisfied up to $A * \bar{\omega}$, $A \gg 1$.
- the spectral weight of the conductivity integrated up to $5\bar{\omega} = 1eV$ enhances below T^*