The spin-fluctuation theory for the high T_c superconductivity. I. The normal state and the pairing.

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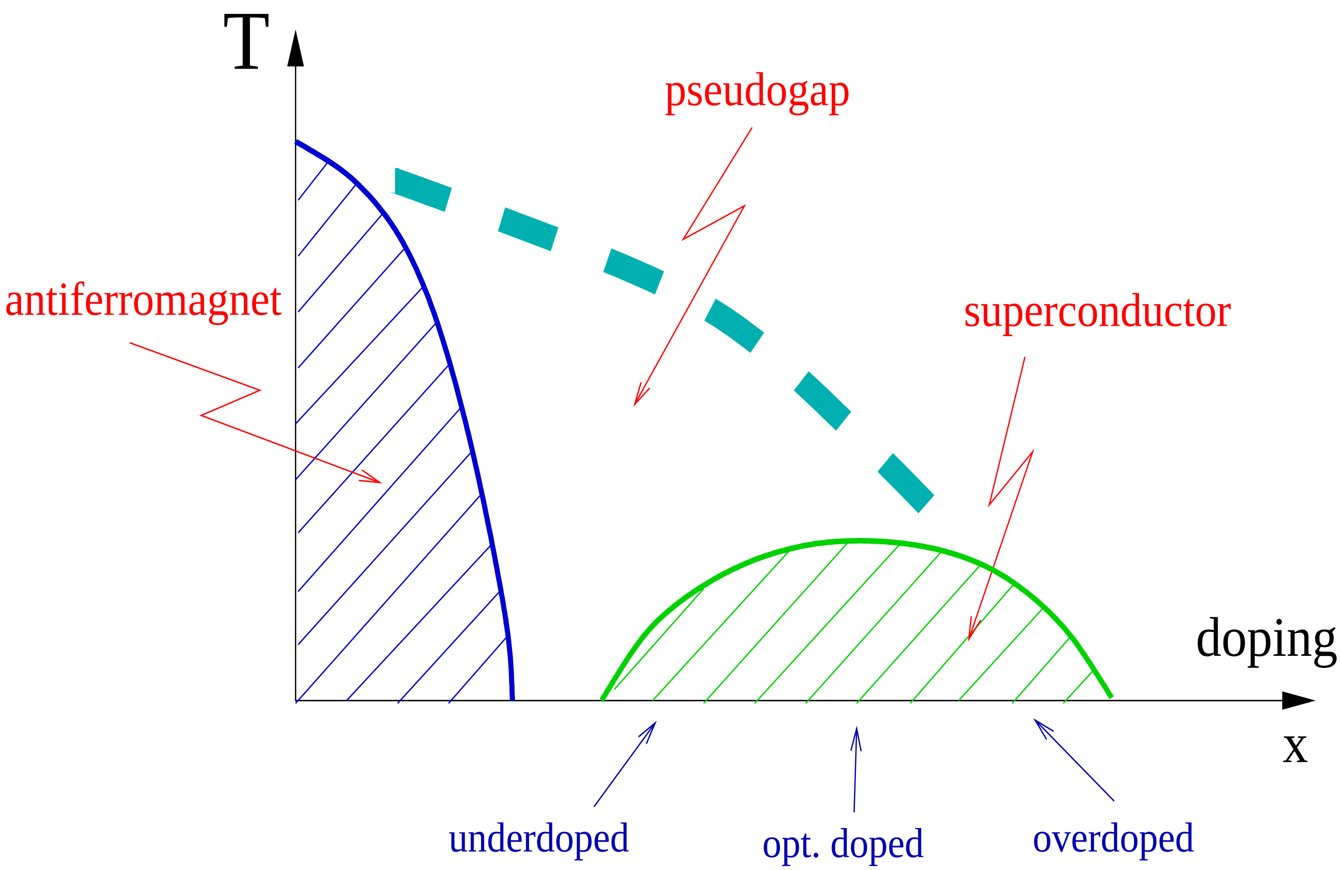
Outline

- The cuprates
- Normal State and Quantum Criticality
- Quantum-Critical Pairing
- Conclusions


T
pseudogap
antiferromagnet
superconductor
doping
x
underdoped
opt. doped
overdoped

Facts:

- antiferromagnetism near half-filling
- $d-$wave symmetry of the superconducting state

Weak coupling theory yields $d-$wave spin-mediated pairing near AFM instability (Scalapino, Pines...)

Why there is still an interest in high T_c?

- non-Fermi liquid behavior in the normal state
- the pseudogap

To which extent superconductivity in the cuprates is the low energy phenomenon?

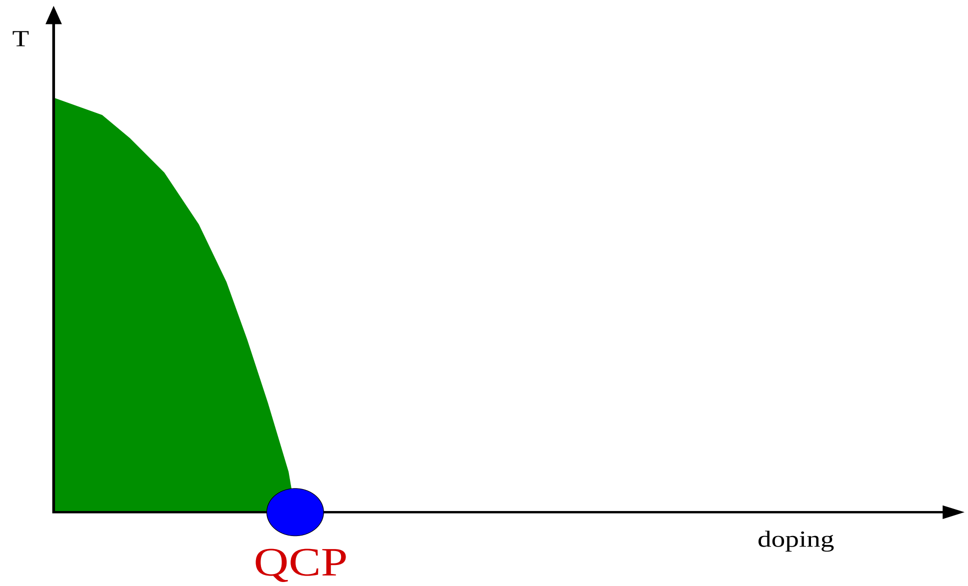
- The Fermi energy $E_F \sim 1eV$ ($p_F \sim 2.5/a$, $V_F \sim 0.8eV * a$)

- The superconducting gap $\Delta \sim 40meV$

$$\Delta/E_F \sim 0.04$$

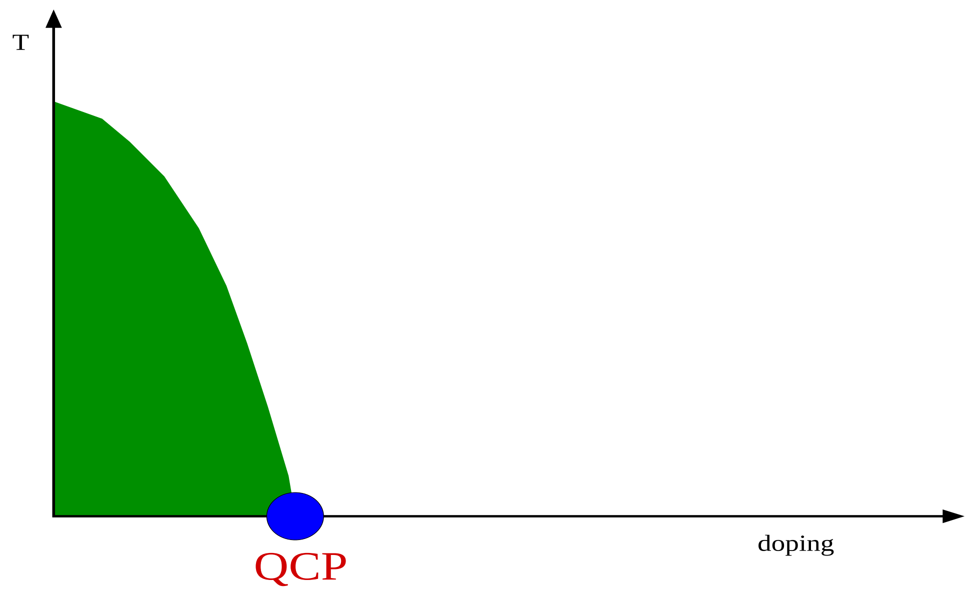
On the other hand, effective Hubbard-type interaction (responsible for antiferromagnetism) is $U \sim 1 - 2eV \sim E_F$.

Antiferromagnetism is the high-energy phenomenon!



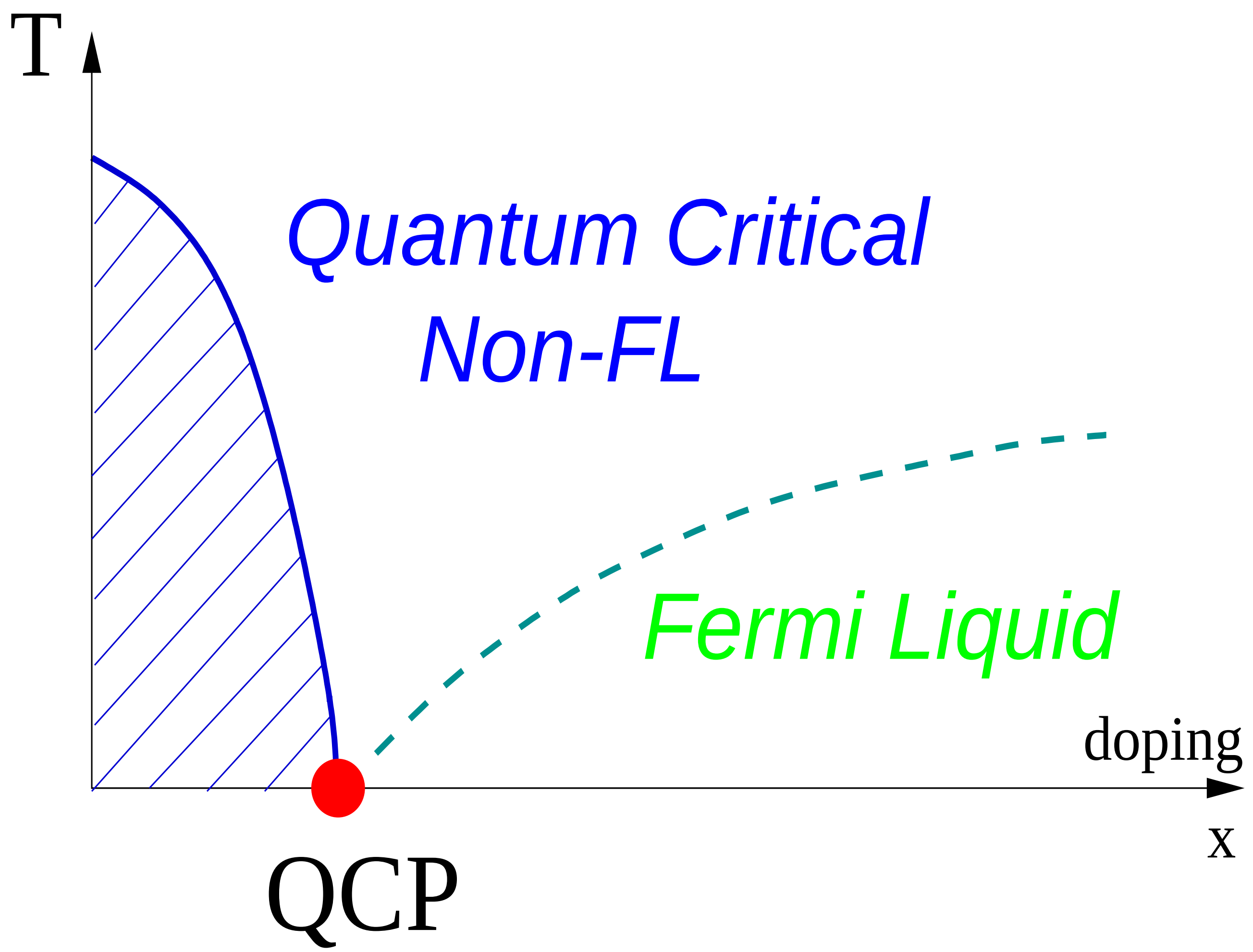
Two different types of approaches to the cuprates

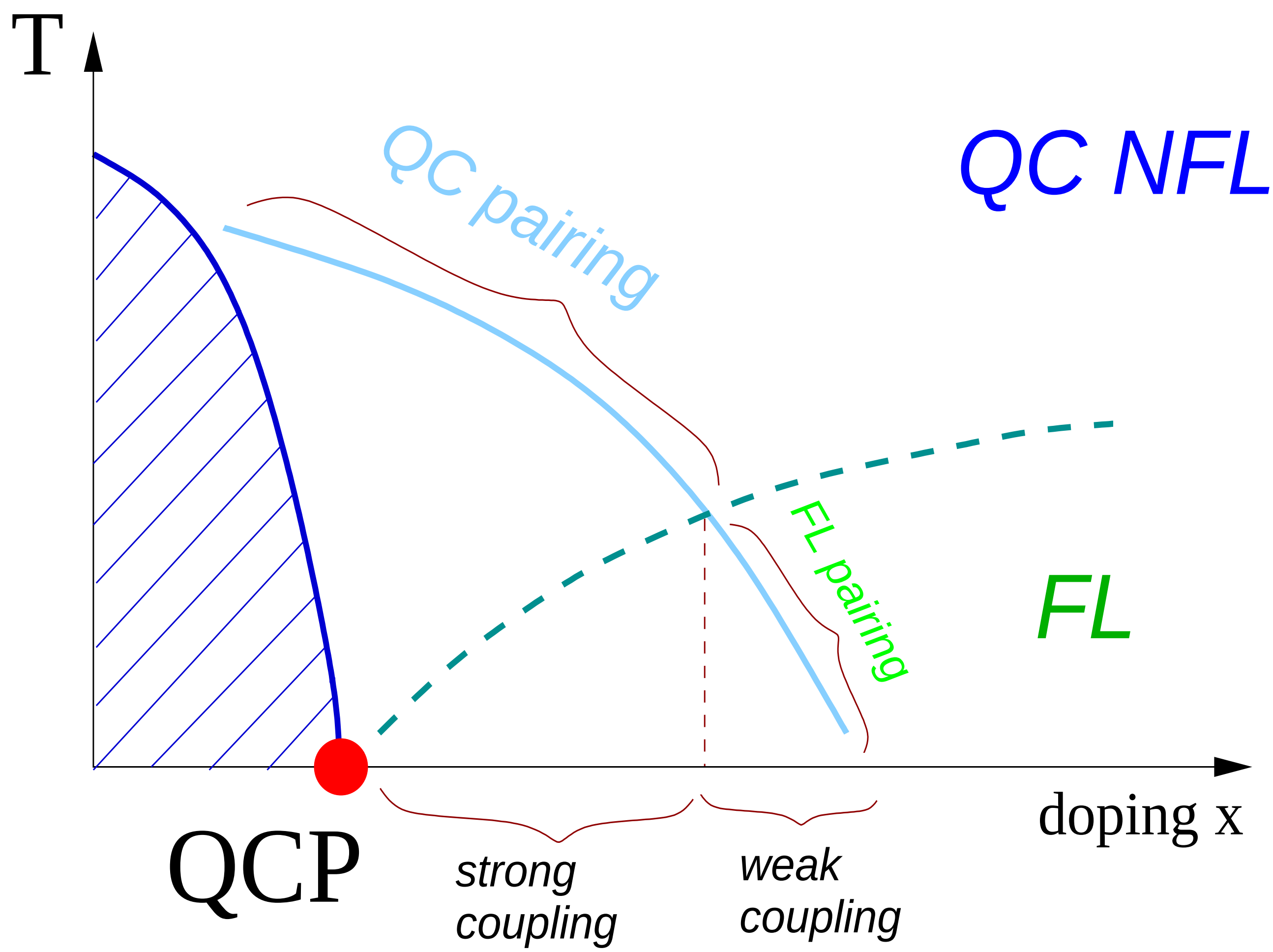
- doping of a quantum antiferromagnet (Mott insulator) (Sachdev, Lee, Anderson)
- approach antiferromagnetism from higher dopings



What happens when we approach antiferromagnetism from the paramagnetic side?

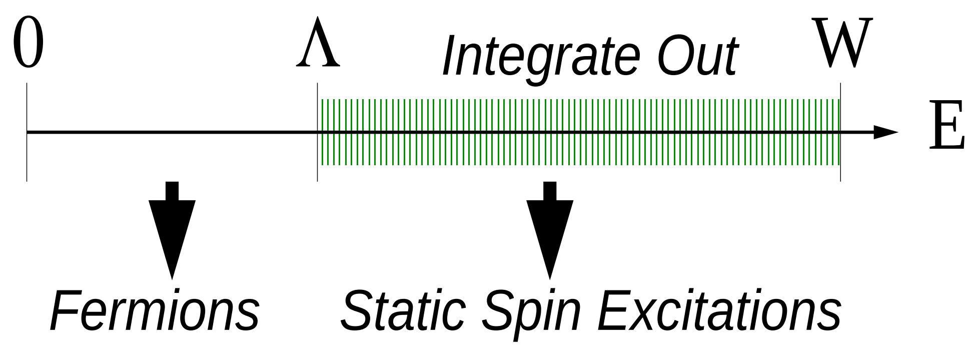
- is there a non FL behavior
- is there a superconductivity
- is there a pseudogap
- is there a secondary critical point at some distance from a magnetic QCP?


T
Quantum Critical
Non-FL
Fermi Liquid
doping
x
QCP

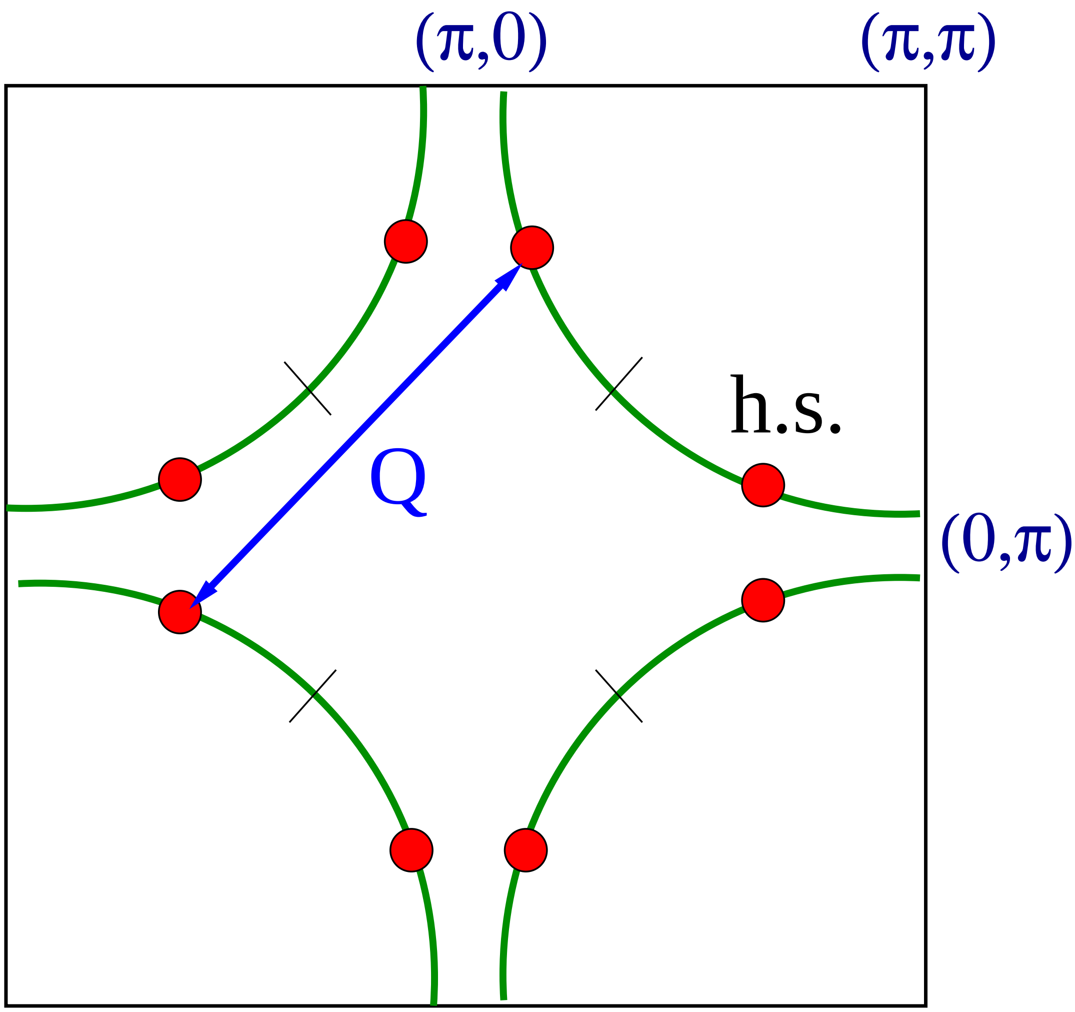

T
QC pairing
QC NFL
FL pairing
FL
doping x
QCP
strong
coupling
weak
coupling

SPIN-FERMION MODEL

Describes the interaction between electrons and their own spin collective degrees of freedom



- Ingredients
  - electrons near the Fermi surface
  - low-energy collective spin excitation
  - spin-fermion coupling (Hubbard U in the RPA)
- Input
  - spin correlation length ξ
  - Fermi surface with hot spots



Spin decay into a particle-hole pair is allowed

$$\mathcal{H} = \mathcal{H}_f + \mathcal{H}_{spin} + \mathcal{H}_{int} \tag{1}$$

$$\mathcal{H}_f = \sum_{\mathbf{k},\alpha} \mathbf{v}_F(\mathbf{k}-\mathbf{k}_F) c^\dagger_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha}$$

$$\mathcal{H}_{spin} = \sum_q \chi_0^{-1}(\mathbf{q}) \vec{S}_q \vec{S}_{-q}$$

$$\mathcal{H}_{int} = g \sum_{\mathbf{q},\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k}+\mathbf{q},\alpha}\, \vec{\sigma}_{\alpha,\beta}\, c_{\mathbf{k},\beta} \cdot \vec{S}_{-\mathbf{q}}$$

The static spin susceptibility is an input

$$\chi(q,\omega) = \frac{\chi_0}{\xi^{-2} + (\mathbf{Q}-\mathbf{q})^2 - (\omega/v_F)^2}, \quad Q \approx (\pi,\pi)$$

The model has two typical energies

- $\bar{\omega} \sim g^2 \chi_0$ – effective interaction (at $\omega > \bar{\omega}$, $\omega > \Sigma(\omega)$)
- $\omega_{sf} \sim (v_F \xi^{-1})^2 / \bar{\omega} \propto \xi^{-2}$ – analog of the Debye frequency

The ratio of the two determines the dimensionless coupling constant

$$\lambda = \left(\frac{\bar{\omega}}{4\omega_{sf}}\right)^{1/2}$$

- $\omega_{sf} > \bar{\omega}$ – weak coupling
- $\omega_{sf} < \bar{\omega}$ – strong coupling

$\lambda \propto \xi$ diverges at the QCP.

Perturbation theory holds in λ^{3-d}

- logarithms in $d = 3$
- powers of λ in $d = 2$

Perturbation theory does not work in $d = 2$ near the QCP.

Near optimal doping, $\omega_{sf} \sim 20 meV$, $\bar{\omega} \sim 200 - 250 meV$, i.e. $\lambda \sim 1.5 - 2$

- for all relevant dopings, we are dealing with the strong coupling problem
- We have to solve simultaneously for fermionic and bosonic self-energies

Strong Coupling Results, Normal State

At $\lambda \geq 1$, spin fluctuations become soft compared to electrons due to a strong decay into particle-hole pairs, and Eliashberg theory becomes applicable.

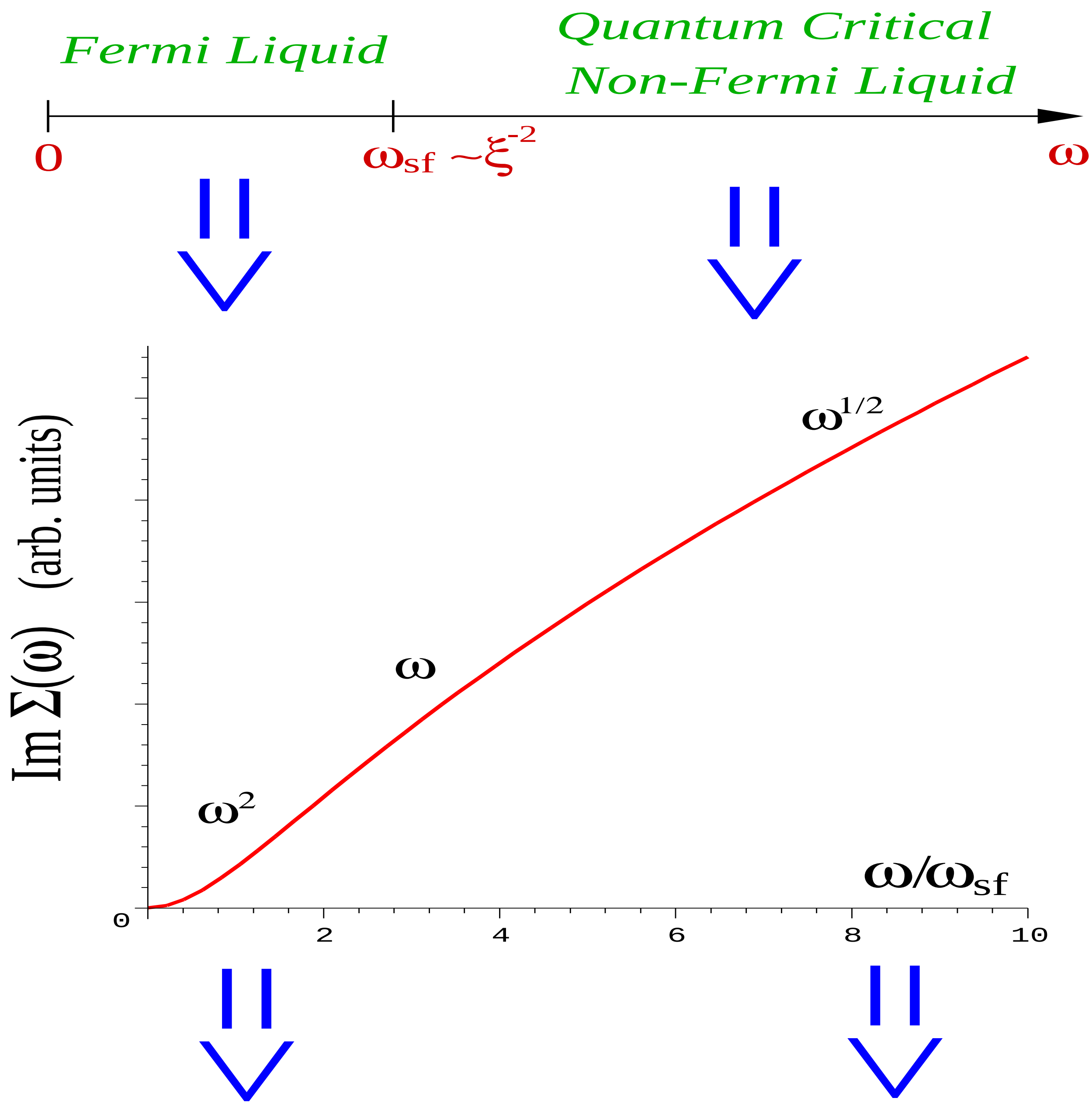
- fermionic self-energy

$$\Sigma(\mathbf{k}, \omega) \approx \Sigma\left(\omega/\omega_{sf}\right)$$

- bosonic self-energy

$$\chi^{-1} \propto 1 + (q-Q)^2\xi^2 - i\omega/\omega_{sf}$$

Fermionic and spin excitations vary at the same scale ω_{sf}.



Fermions: FL
Spin excitations: static

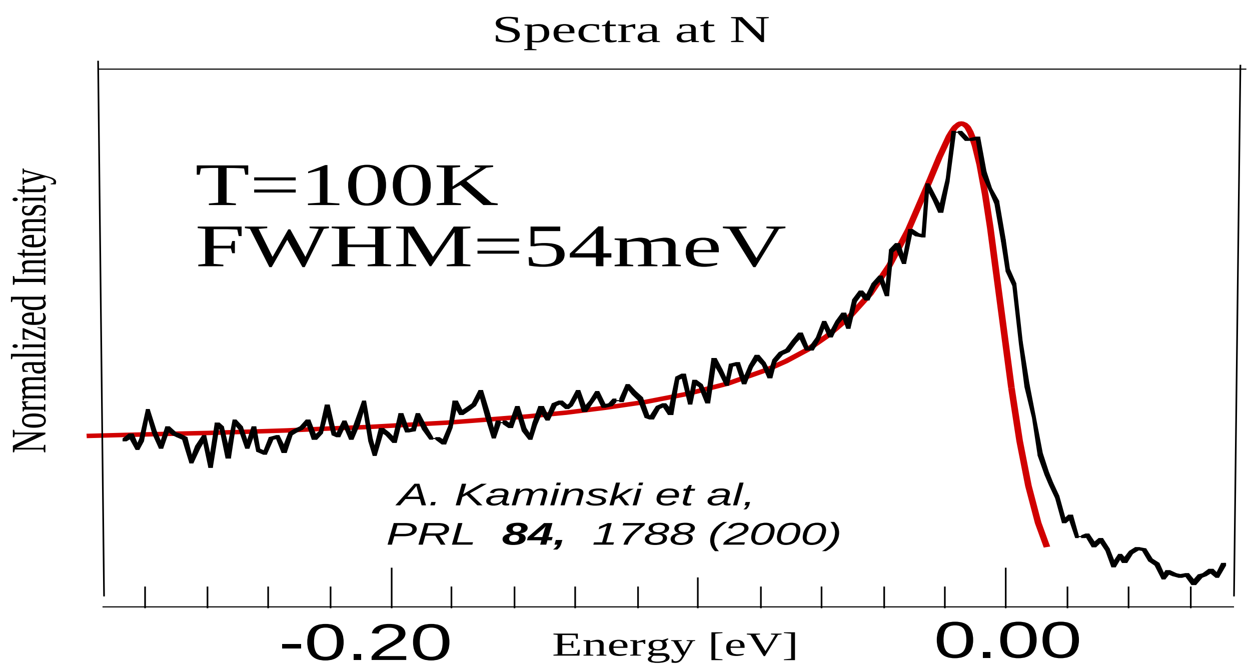
$\chi^{-1}(q,\omega) \sim q^2 + \xi^2$

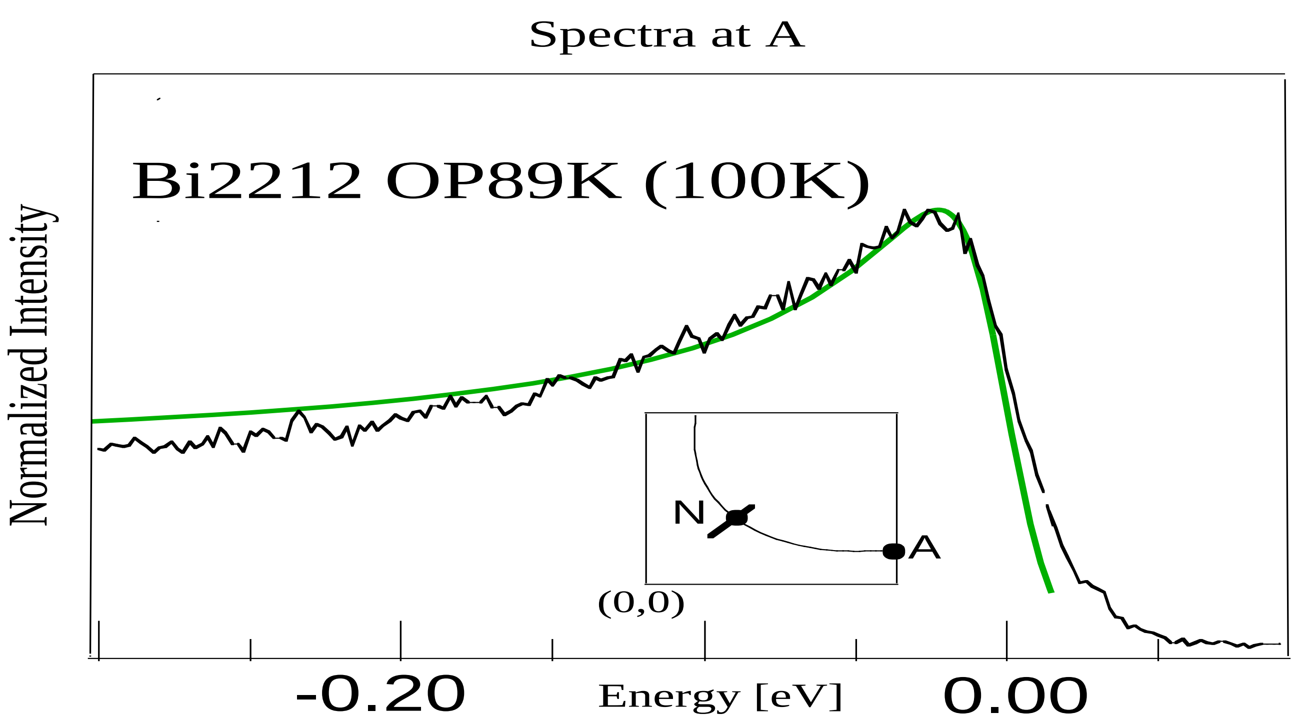
Fermions: QC NFL
Spin excitations: relaxational

$\chi^{-1}(q,\omega) \sim q^2 + i\,\omega/\omega$

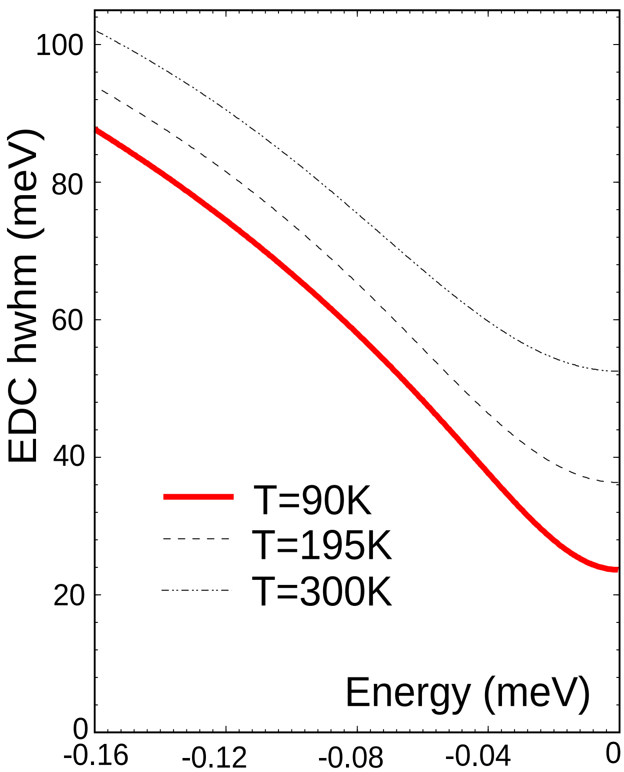
$G^{-1}(\omega) \sim \sqrt{\omega}$

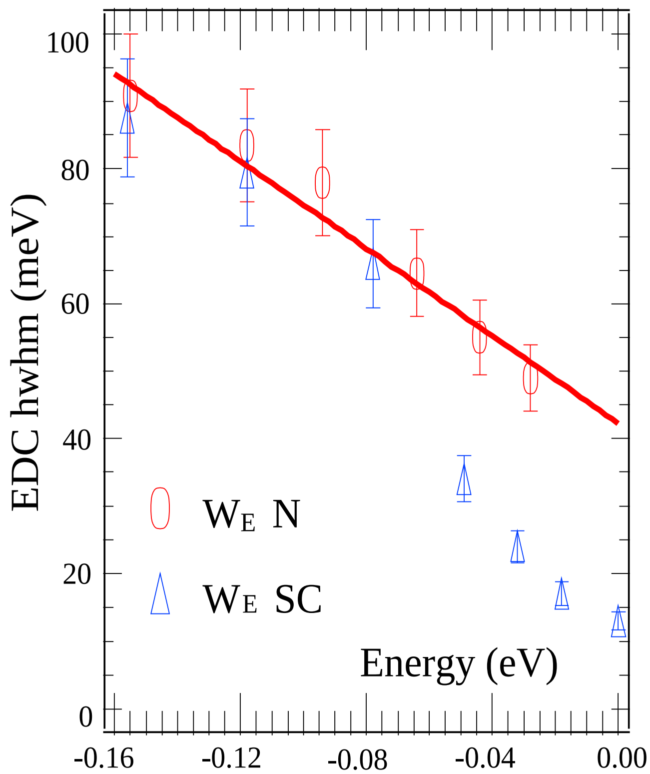
The EDC photoemission intensity, optimal doping



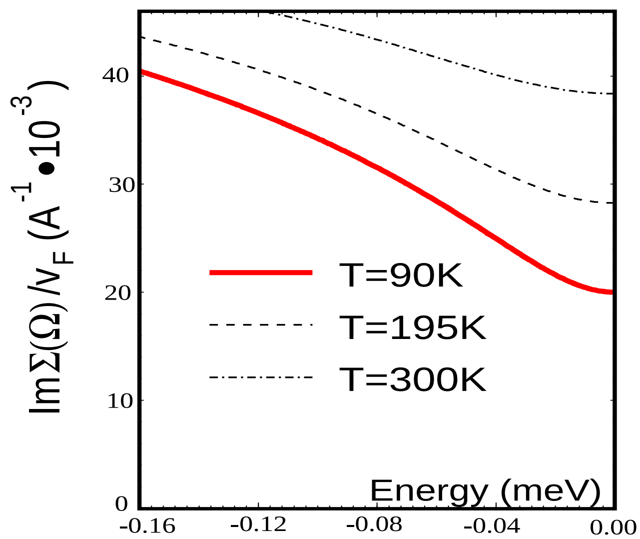


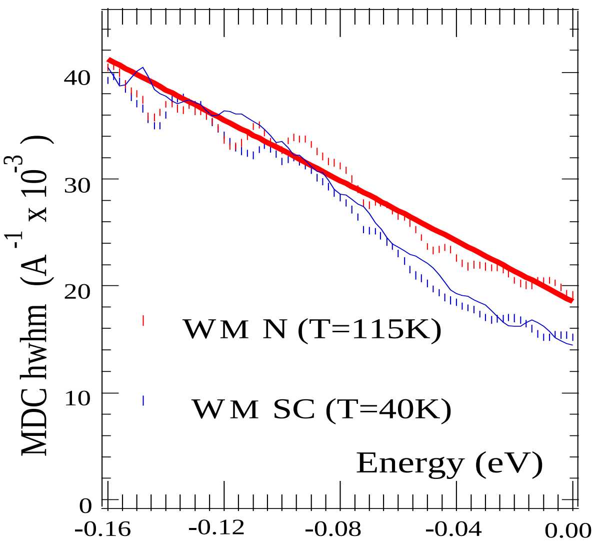
The fermionic self-energy $\Sigma(\omega)$ extracted from the EDC photoemission data, optimal doping

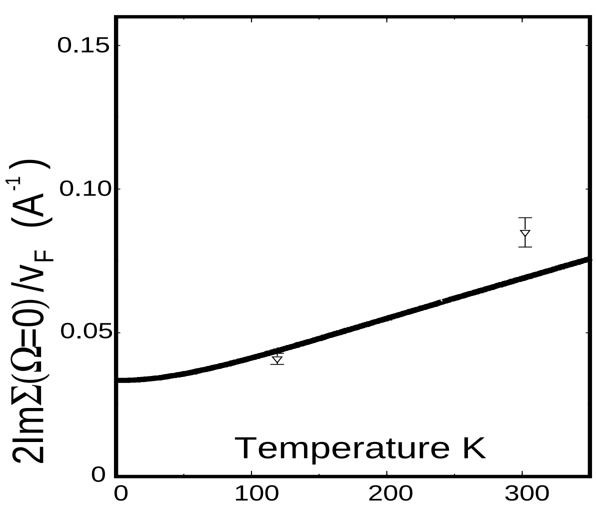


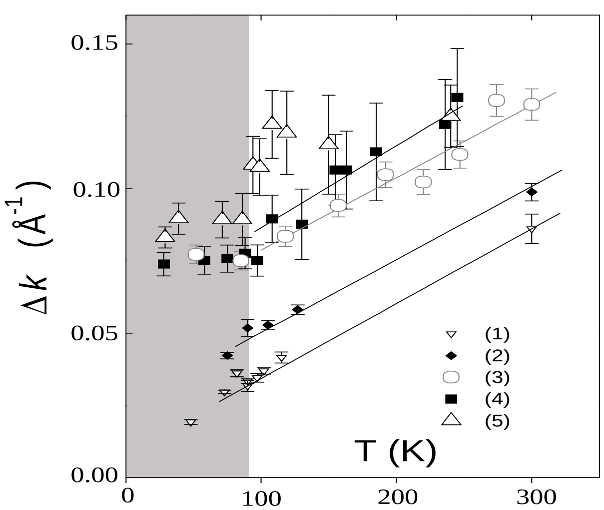


The fermionic self-energy $\Sigma(\omega, T)$ extracted from the MDC photoemission data, optimal doping

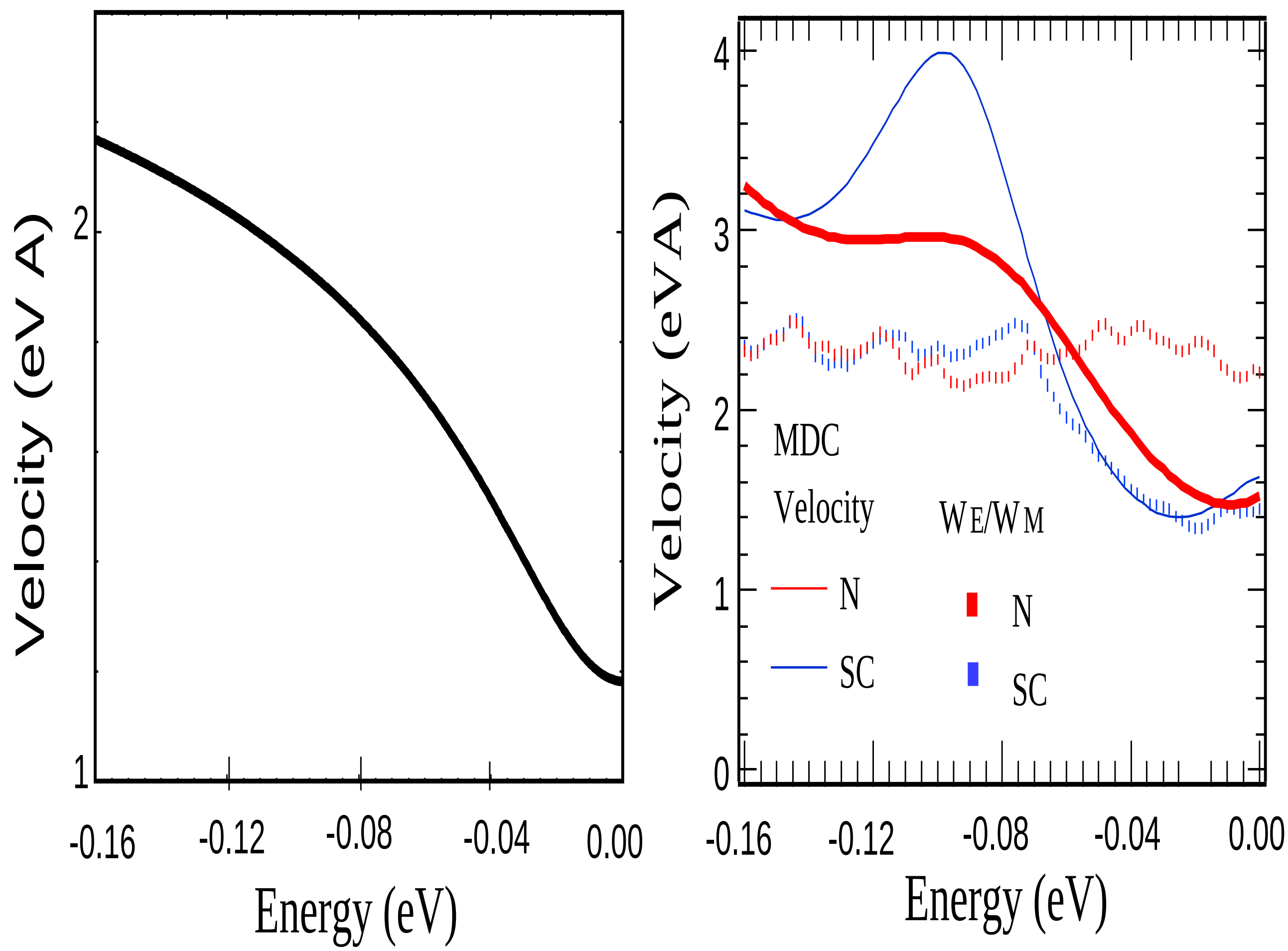




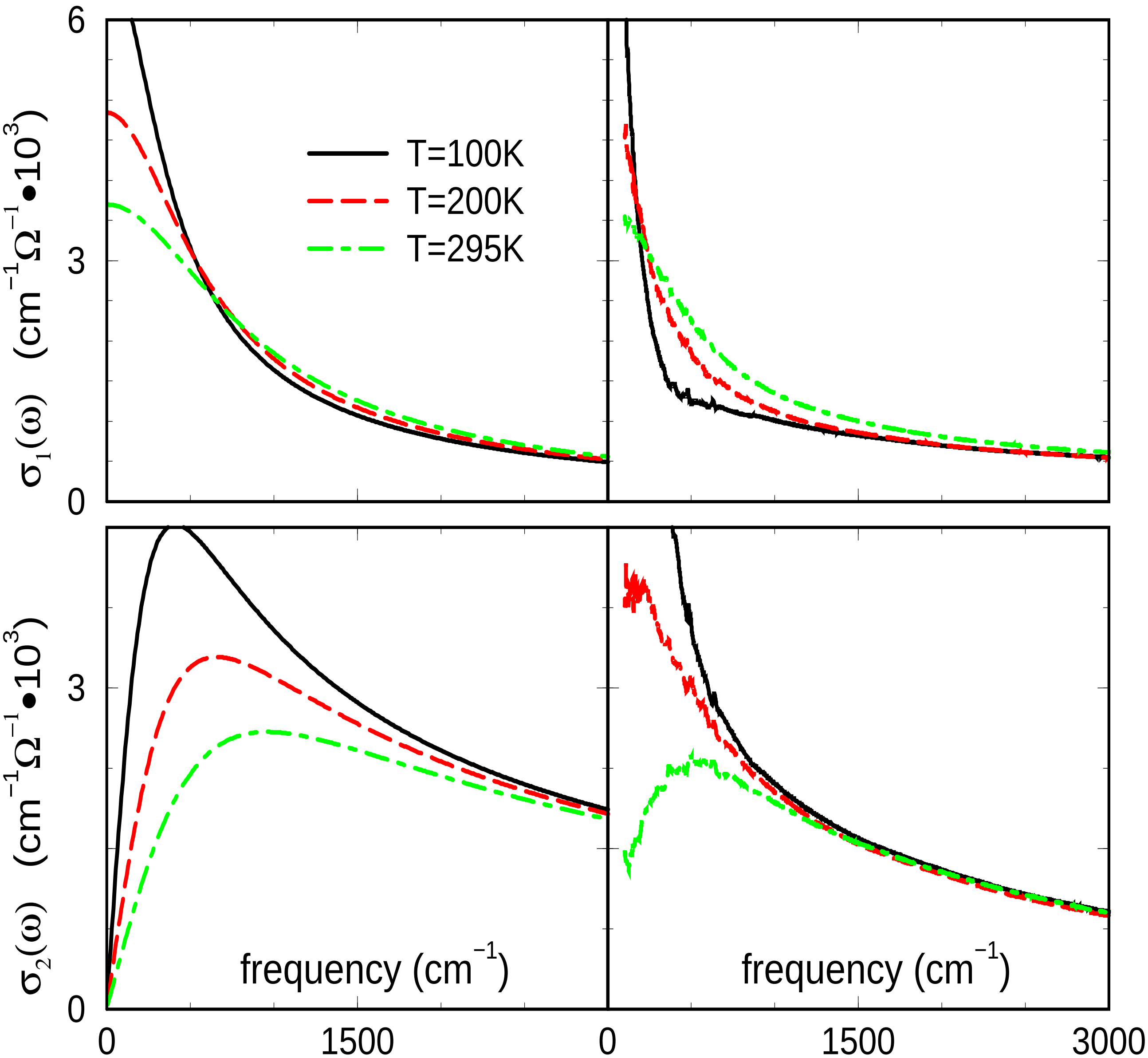




The energy dependence of the quasiparticle velocity $v_{eff} = v_F/(1+\lambda(\omega))$ extracted from the MDC photoemission data, optimal doping

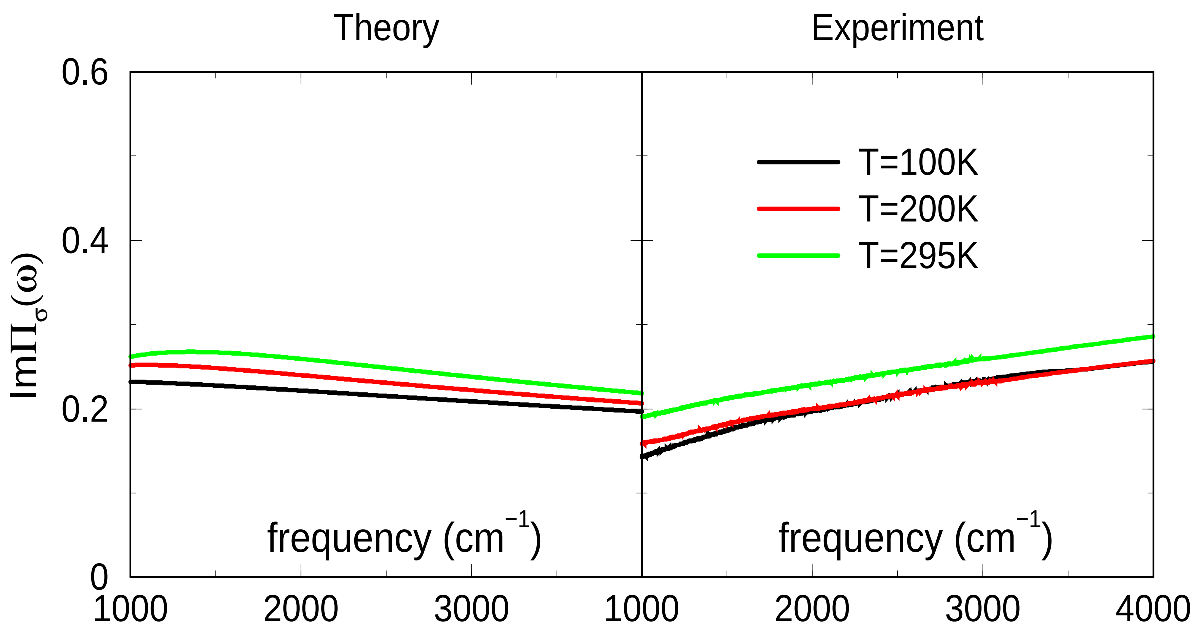


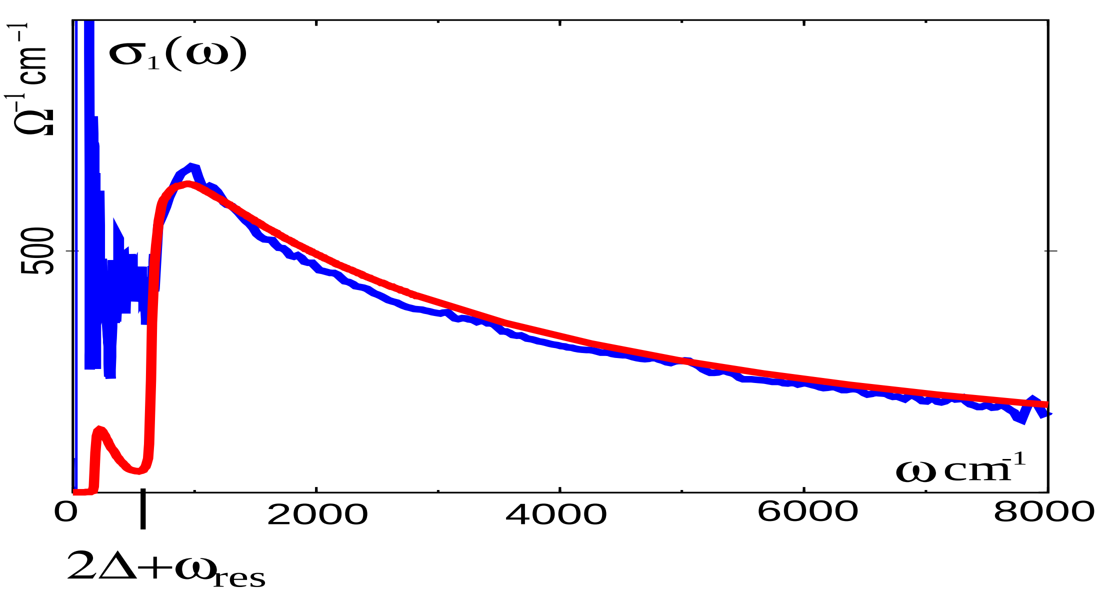
Real and imaginary part of optical conductivity at various temperatures, optimal doping

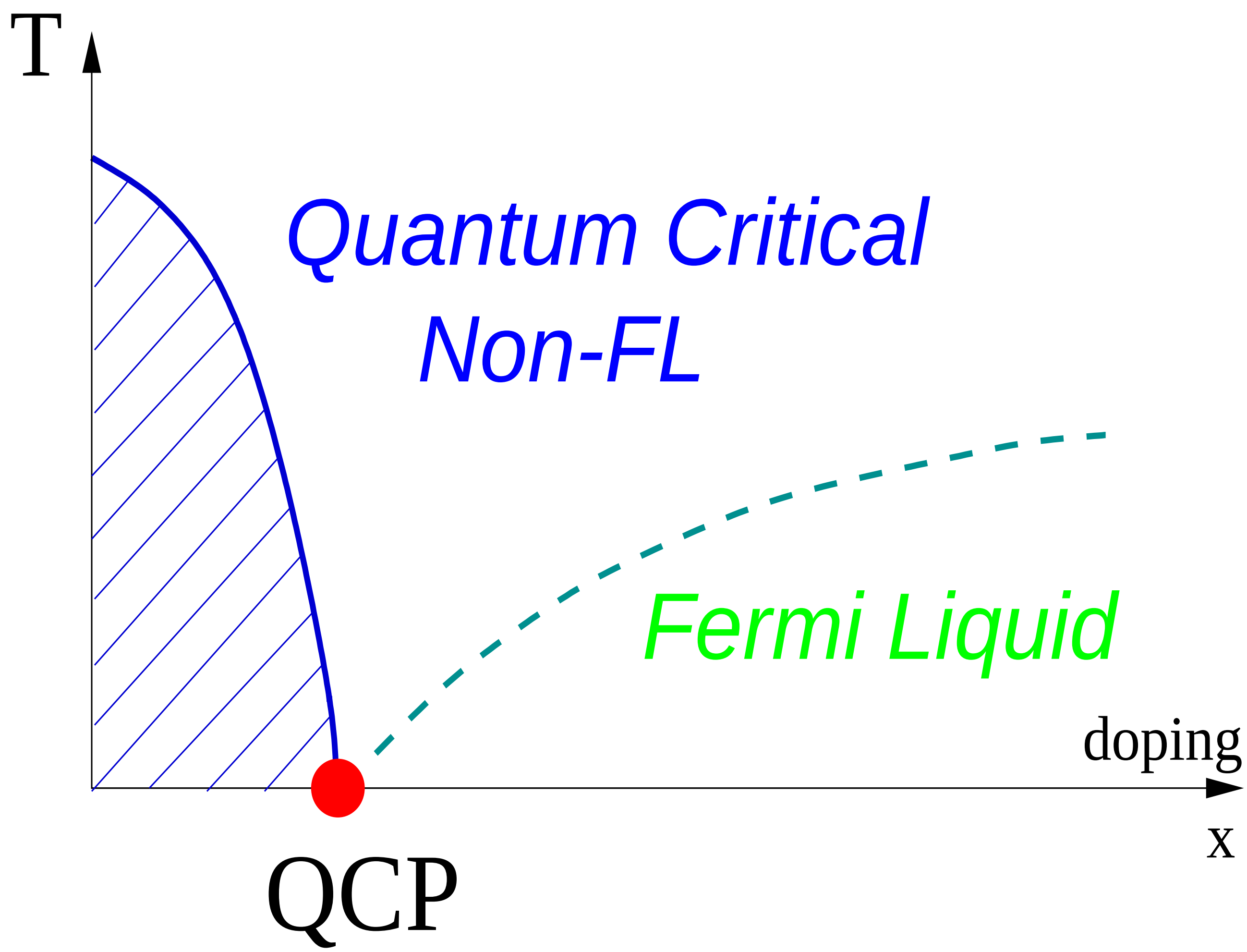


The imaginary part of the current-current polarization operator, optimal doping

$$\sigma(\omega) = \frac{\omega_{pl}^2}{4\pi} \frac{i\Pi(\omega)}{\omega}$$



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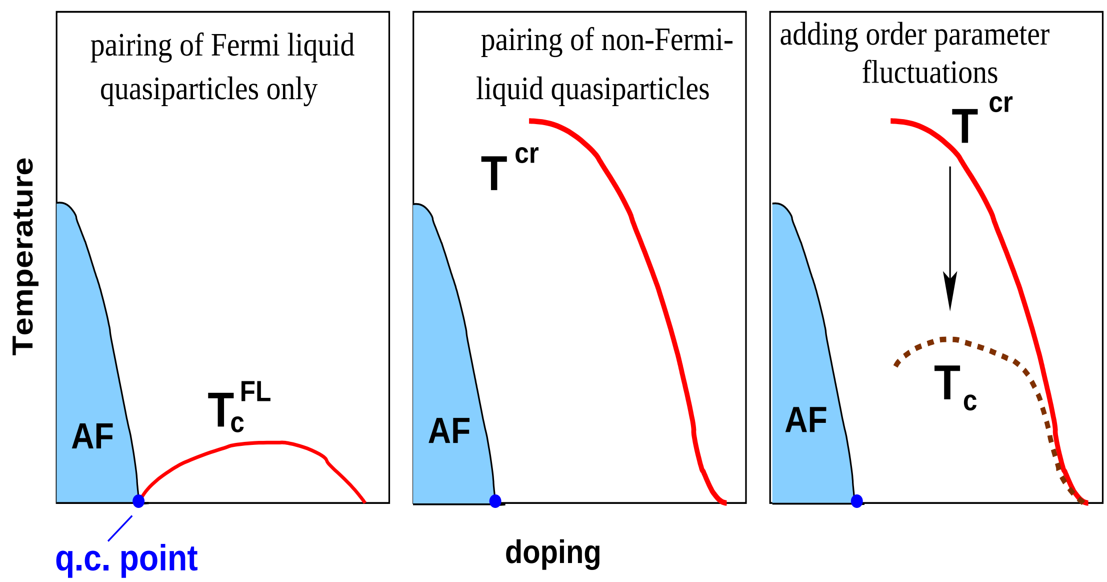
Pairing problem

- Spin-mediated pairing yields attraction in $d_{x^2-y^2}$ channel
  - Scalapino, Pines, Schrieffer ...

Which of the two scales, $\omega_{sf} \propto \xi^{-2}$ or $\bar{\omega} \sim 2J$ determines the pairing instability?



McMillan -type reasoning: $T_c \sim \omega_{sf}$

- at $\omega < \omega_{sf}$, λ is reduced by a mass renormalization $\Longrightarrow \ \lambda_{eff} = \frac{\lambda}{1+\lambda} = O(1)$
- pairing interaction decreases above ω_{sf}

At $\omega < \omega_{sf}$, the system behaves as a Fermi liquid with an effective pairing coupling $O(1)$.

$$T_c \sim \omega_D \exp\left[-\frac{1+\lambda}{\lambda}\right]$$

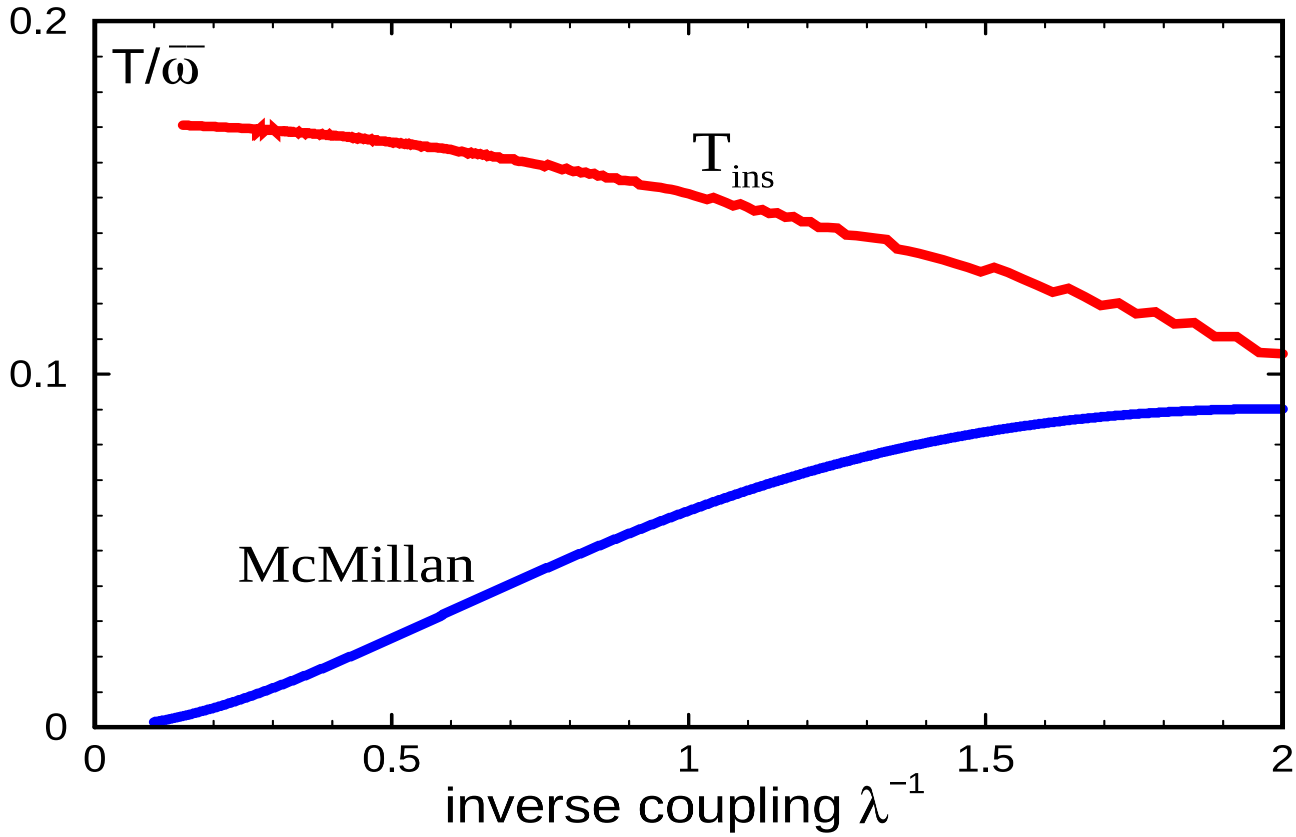
Alternative reasoning: $T_c \sim \bar{\omega}$

- above ω_{sf}, λ becomes $\lambda(\omega)$
- the reduction of $\lambda(\omega)$ reduces the mass renormalization
- $\lambda_{eff} \approx 1$ up to $\omega \sim \bar{\omega}$

A novel, universal, non BCS pairing problem: NFL fermions with attaction due to an exchange of gapless spin collective modes.

Numerical and analytical analysis:

A linearized $d-$wave gap equation has a solution at $T_{ins} \sim \bar{\omega}$



Consider $\omega_{sf} = 0$.

The linearized equation for the $d-$wave pairing vertex is

$$\Phi_k(\omega) = \frac{\pi T}{2}\epsilon_k \sum_{\omega'} \frac{\Phi_k(\omega')}{\sqrt{|\omega'|\,|\omega' - \omega|}} \frac{1}{1 + \sqrt{|\omega'|/\bar{\omega}}} + \Phi_0$$

$\epsilon_k = 1$ at a hot spot, and decreases away from a hot spot

The kernel is $O\left(\frac{1}{\omega}\right)$ as in BCS theory, but now

- $|\omega|^{-1/2}$ comes from fermionic self-energy
- $|\omega|^{-1/2}$ comes from gapless collective mode

If a solution exists at $\Phi_0 = 0$, $T_{ins} \sim \bar{\omega}$, but no guarantee that this solution does exist.

Suppose $\epsilon_k \ll 1$ (far away from a hot spot)

We can perturbatively sum logarithms, as in BCS theory.

In BCS theory

$$\chi_{pp}(T) = \frac{\Phi(T)}{\Phi_0} = \frac{1}{\log T/T_c}$$

- the susceptibility diverges at T_c
- below T_c, χ_{pp} is negative

In our case

$$\chi_{pp} = \left(\frac{\bar{\omega}}{\omega}\right)^{\epsilon_k}, \qquad T = 0$$

- the susceptibility remains positive, no instability

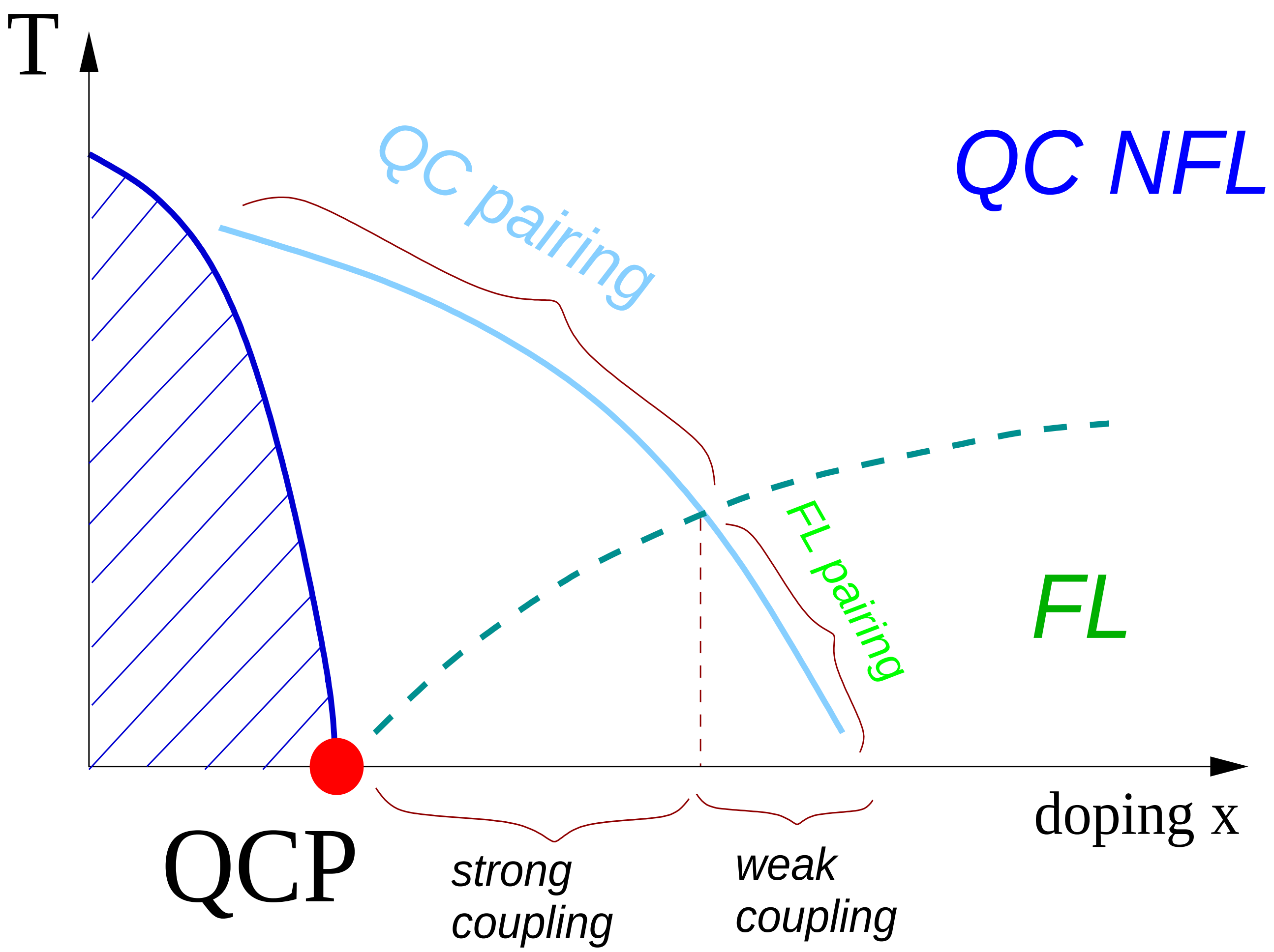
Near a hot spot, $\epsilon_k \approx 1$

- threshold at $\epsilon_k = \epsilon_{th} = 0.221$
- instability at $\epsilon_k > \epsilon_{th}$.

$$T_c \sim e^{-A/(\epsilon - \epsilon_{th})}, \quad A \approx 3$$

- the non BCS solution exists only in a finite range of k

At large λ, $\Delta(k)$ becomes peaked at hot spots


T
QC NFL
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QCP
doping x
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weak
coupling

CONCLUSIONS

- strong interaction between fermions and their own low-energy spin collective modes yields:
  - non-Fermi liquid, QC behavior in the normal state between $\omega_{sf} \propto \xi^{-2}$ and noncritical $\bar{\omega}$
  - NFL behavior is consistent with photoemission and conductivity measurements
  - near the QCP, the system undergoes a non BCS pairing instability at $T_{ins} \sim \bar{\omega}$