Final Exam

Physics 752
Many body problems in solid state physics
Fall, 2002

1 Express the density of a Fermi gas via the integral of the Green's function. Obtain the relation between the Fermi momentum and the density for three and two dimensions.

2. Consider a system of electrons that interact with Einstein phonons (phonon energy is equal to ω_D and is independent of momentum). Obtain real and imaginary part of the fermionic self-energy for fermions near the Fermi surface (approximate $\int d^3k$ by $N_0 \int d\epsilon_k$ where N_0 is a constant).

3. Obtain the leading small-momentum dependence of a damping rare of a spin wave in a 3D ferromagnet at a finite T.

4. Check whether the spin-wave in an antiferromagnet at $T = 0$ can have a finite decay rare (use the analogy with a superfluid Bose liquid).

5. Obtain the Green's function of a free Fermion gas at $T = 0$ in mixed coordinate/frequency representation (i.e., $G(\omega, r_1 - r_2)$) at $p_F|r_1 - r_2| \gg 1$. Do integration in two ways: (i) integrate explicitly over $\epsilon_k = k^2/(2m) - E_F$, and (ii) approximate $\epsilon_k \approx v_F(k - k_F)$ and then integrate over k. Compare the results.

6. Extra problem. Consider a Fermi gas + an extra localized spin $\vec{S}$ at $r = 0$. Assume that this localized spin is interacting with the electron spin density as

$$H_{int} = J \int d^3r S^i \delta(r) \phi^\dagger(r) \sigma^i \phi(r) \tag{1}$$

where σ^i are the Pauli matrices ($i = x, y, z$). Find the polarization of electrons

$$\sigma^i(r) = < \phi^\dagger(r) \phi(r) > \tag{2}$$

at large distances r, to first order in J. Show that the polarization oscillates as a function of coordinates. Approximate fermionic dispersion as $\epsilon_k = v_F(k - k_F)$.