

MIDTERM 1

**Physics 311  
Mechanics  
Spring, 2003**

1. A particle moves in the region  $x > 0$  subject to the potential energy

$$U(x) = U_0 \left( \frac{a}{x} + \frac{x}{a} \right) \quad (1)$$

where  $a > 0$ ,  $U_0 > 0$ . Locate all stable equilibrium points and obtain the frequencies of small oscillations about these points.

2. For a particle moving in a plane under the influence of a central potential energy  $V(r)$ , find the energy  $E$  as a function of  $r$ ,  $\theta$ ,  $p_r$  and  $p_\theta$  ( $p_r$  and  $p_\theta$  are momenta). Show that the Lagrange's equations of motion are equivalent to

$$\frac{\partial E}{\partial p_\alpha} = \frac{dq_\alpha}{dt} \quad (2)$$

$$\frac{\partial E}{\partial q_\alpha} = -\frac{dp_\alpha}{dt} \quad (3)$$

where  $q_1 = r$ ,  $q_2 = \theta$ ,  $p_1 = p_r$ ,  $p_2 = p_\theta$ .

- 3.

Extra:

Period of oscillations is given by

$$T = \frac{\pi\sqrt{2m}}{\alpha\sqrt{|E|}}, \quad E < 0, \alpha > 0 \quad (4)$$

Find the potential energy  $U(x) = U(-x)$ .