Name Solutions

Exam #1
Physics 247
October 4, 2000

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. An atom moving in 1-D optical molasses experiences an acceleration

$$a = \frac{-\gamma v}{1 + (v/v_c)^2}$$

where γ and v_c are positive constants. The atom initially has velocity v_c.

(a) (10 pts) Use dimensional analysis to estimate the distance required to stop the atom.

$$[\gamma] = s^{-1}$$

$$[v_c] = \frac{m}{s}$$

$$\therefore \quad \boxed{\Delta x = \frac{v_c}{\gamma}}$$

(b) (15 pts) Now use your knowledge of 1-D motion to give an exact answer. (Hint: $a = vdv/dx$)

$$v\frac{dv}{dx} = \frac{-\gamma v}{1 + (\frac{v}{v_c})^2}$$

$$\left(1 + \frac{v^2}{v_c^2}\right) dv = -\gamma\, dx$$

$$\int_{v_c}^{0} \left(1 + \frac{v^2}{v_c^2}\right) dv = -\gamma \int_0^x dx$$

$$\left[v + \frac{1}{3}\frac{v^3}{v_c^2}\right]_{v_c}^{0} = -\gamma x$$

$$x = -\frac{1}{\gamma}\left[0 + 0 - v_c - \frac{1}{3}\frac{v_c^3}{v_c^2}\right]$$

$$\boxed{x = \frac{4}{3}\frac{v_c}{\gamma}}$$

2. A pitcher (whose feet are at the origin of a coordinate system) throws a curveball in the $\hat{x}$ direction from a height $z = 4$ ft toward home plate (located at $x = 60$ ft, $y = 0$, $z = 0$) at a speed of 120 ft/s. Due to the "Magnus effect", the spinning ball experiences a horizontal (y) acceleration of $a_y = 4$ ft/s^2. Neglect wind resistance. (Note $g = 32$ ft/s^2)

   (a) 5 pts) Write down the acceleration, initial velocity, and initial position vectors.

$$\vec{a} = 4 \frac{ft}{s^2} \hat{j} - 32 \frac{ft}{s^2} \hat{k}$$
$$\vec{v}_0 = 120 \frac{ft}{s} \hat{i}$$
$$\vec{x}_0 = 60 \text{ ft } \hat{i}$$

   (b) 10 pts) Where does the ball hit the ground?

$$\Delta z = 4 \text{ ft} = \frac{1}{2}\left(32 \frac{ft}{s^2}\right) t^2$$
$$\Rightarrow \quad t^2 = \frac{4}{16} s^2 = \frac{1}{4} s^2$$
$$\Rightarrow \quad t = \frac{1}{2} s$$
$$\therefore \quad \Delta x = 120 \frac{ft}{s} \cdot \frac{1}{2} s = 60 \text{ ft}$$
$$\Delta y = \frac{1}{2}\left(4 \frac{ft}{s^2}\right)\left(\frac{1}{2} s\right)^2 = \frac{1}{2} \text{ ft.}$$

$\therefore$ Hits ground at $\vec{x}_f = (60 \text{ ft})\hat{i} + \left(\frac{1}{2} \text{ ft}\right)\hat{j}$

or $\frac{1}{2}$ ft to left of home plate.

   (c) 10 pts) What is its velocity when it hits the ground?

$$\vec{v}_f = \left(120 \frac{ft}{s}\right)\hat{i} + 4 \frac{ft}{s^2} \cdot \frac{1}{2} s \,\hat{j} - 32 \frac{ft}{s^2} \cdot \frac{1}{2} s \,\hat{k}$$
$$\boxed{\vec{v}_f = 120 \frac{ft}{s} \hat{i} + 2 \frac{ft}{s} \hat{j} - 16 \frac{ft}{s} \hat{k}}$$

   (d) 0 pts) What does the umpire yell?

Ball !

3. You carry a heavy 500N box at a height of 1.5 m onto an elevator on the ground floor. Forgetting to set it down, you ask your friend to hit the 5th floor button. Assume $g=10 m/s^2$.

(a) 10 pts) Assuming a uniform acceleration for the elevator of $2m/s^2$, how much heavier does the box feel as the elevator starts to move?

$$\Delta W = m a$$

$$m = \frac{500\,N}{10\,\frac{m}{s^2}} = 50\,kg$$

$$a = 2\,\frac{m}{s^2}.$$

$$\therefore \boxed{\Delta W = 100\,N}$$

(b) 10 pts) If you can't handle the load, and you drop the box as soon as the elevator starts to move, how long does the box take to hit the elevator floor?

$$1.5\,m = \frac{1}{2} \cdot 12\,\frac{m}{s^2}\,t^2$$

$$\therefore \quad t^2 = \frac{3}{2}\,m \cdot \frac{1}{6\,\frac{m}{s^2}} = \frac{1}{4}\,s^2$$

$$\Rightarrow \boxed{t = \frac{1}{2}\,s}$$

(c) 5 pts) How far has it travelled relative to the elevator shaft when it hits the floor?

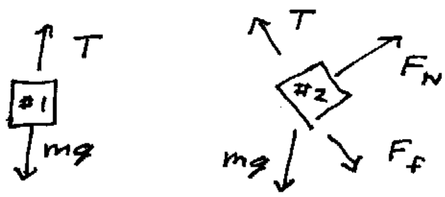
$$\Delta z = \frac{1}{2} \cdot 10\,\frac{m}{s^2} \cdot \left(\frac{1}{2}\,s\right)^2$$

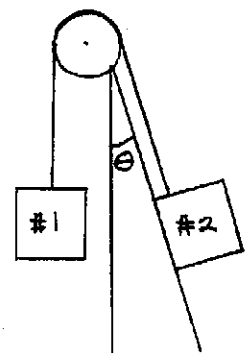
$$= 5\,\frac{m}{s^2} \cdot \frac{1}{4}\,s^2$$

$$\boxed{\Delta z = 1.25\,m}$$

4. Two 1 kg blocks are connected by a cord and pulley, as shown. Block #1 hangs freely, while block #2 rests on an incline of angle θ. The coefficient of static friction $\mu_s = 0.01$.

(a) 5 pts) Draw a free-body diagram for both blocks.

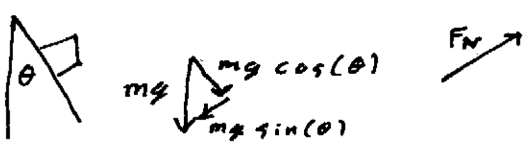




(b) 5 pts) Assuming the blocks are not moving, find the tension in the cord.

$T = mg$, because #1 is not accelerating

(c) 10 pts) Sum the forces on block #2 in the direction parallel to the incline, and set them equal to zero. As you know, this determines the angle θ at which the block will start to slide. Your answer should involve only θ and μ_s. Don't solve it for θ (yet).



$\Rightarrow F_N = mg \sin(\theta)$

$\Rightarrow F_f = \mu_s mg \sin(\theta)$

$$\overset{mg}{T} - mg\cos(\theta) - \mu_s mg \sin(\theta) = 0$$

$$\therefore \quad mg - mg\cos(\theta) - \mu_s mg \sin(\theta) = 0$$

$$\Rightarrow \boxed{1 - \cos(\theta) - \mu_s \sin(\theta) = 0}$$

(d) 5 pts) Since μ_s is very small, the angle θ must also be small and we can make the approximations $\sin\theta \approx \theta$, and $\cos\theta \approx 1 - \theta^2/2$. Using these approximations, solve for θ.

$$1 - \left(1 - \frac{\theta^2}{2}\right) - \mu_s \theta = 0$$

$$1 - 1 + \frac{\theta^2}{2} - \mu_s \theta = 0$$

$$\frac{\theta^2}{2} = \mu_s \theta \quad \Rightarrow \boxed{\theta = 2\mu_s = 0.02}$$