Name Solutions

Exam #1

Physics 247

October 1, 2003

Each problem is worth 25 points

Problem	Score
1	25
2	25
3	25
4	25
Total	100

Curve = $60 + (\text{raw} - 33) \cdot \frac{20}{31}$

1. Modern atomic "fountain" clocks work by tossing atoms up and then counting their internal "clicks" until they return to where they began. A cold Cs atom inside an atomic clock is launched upwards with a speed of 6 m/s. (Take $g = 10$ m/s^2).

(a) At least how tall is the clock?

$$v = v_0 - gt = 0 \qquad t = \frac{v_0}{g}$$

$$x = v_0 t - \frac{1}{2} g t^2 = \frac{v_0^2}{g} - \frac{1}{2} g \frac{v_0^2}{g^2} = \frac{1}{2} \frac{v_0^2}{g}$$

$$= \frac{1}{2} \frac{(6\,m/s)^2}{10\,m/s^2} = 1.8\,m$$

(b) How slow must the atom be moving in the horizontal direction so that it when it returns it comes within 10 cm of its original launch point?

$$2 v_y t = y$$

$$v_y = \frac{y}{2t} = \frac{y g}{2 v_0} = \frac{(0.1\,m)(10\,m/s^2)}{2 \; 6\,m/s} = \frac{1}{12}\,m/s$$

2. A particle of mass m moves in a force field $\vec{f} = -kx\hat{\imath}$. Starting at $x = 0$ with initial velocity $v_0\hat{\imath}$, how far does it move before stopping?

(a) Use dimensional analysis to estimate the answer.

$$x = \left(\frac{k}{m}\right)^p (v_0)^q \qquad [k] = \frac{kg}{s^2}$$

$$m = \left(\frac{1}{s^2}\right)^p \left(\frac{m}{s}\right)^q \Rightarrow q = 1 \quad p = -\tfrac{1}{2}$$

$$x = \sqrt{\frac{m}{k}}\, v_0$$

(b) Now give an exact answer.

$$v\frac{dv}{dx} = -\frac{k}{m}x$$

$$\frac{1}{2}(v^2 - v_0^2) = -\frac{k}{m}\frac{x^2}{2}$$

($v \to 0$)

$$x = \sqrt{\frac{m}{k}}\, v_0$$

3. A particle of mass 10^{-20} kg and initial velocity $(10\hat{\imath} + 6\hat{k})$ m/s enters a region with a uniform magnetic field $\vec{B} = 10$ T $\hat{k}$. The only force on the particle is the Lorentz force $F = q\vec{v} \times \vec{B}$, and the charge on the particle is $q = 10^{-15}$ C. All of these units are standard SI units, i.e. 1 N= 1 C-T-m/s.

9 (a) What is the z-component of the velocity as a function of time?

no $\vec{z}$ force so $v_z = 6$ m/s

9 (b) The particle traces a path which is a helix centered around the z-axis. What is the radius of the circular motion that occurs in the x-y plane?

$$F = \frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB} = \frac{10^{-20}\,\text{kg}\ 10\,\text{m/s}}{10^{-15}\,\text{C}\ 10\,\text{T}} = 10^{-5}\,\text{m}$$

5 (c) If this particle were less massive, would the radius of the helix be bigger or smaller?

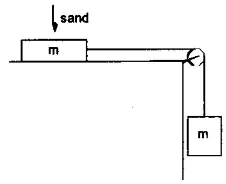
$r \propto m \Rightarrow$ smaller

2 (d) How many times heavier or lighter would the particle have to be to trace a helix with the same diameter as DNA? An answer within a factor of ten of the correct result will be fine.

DNA ~ 10nm = 10^{-8} m

$\therefore$ 1,000 × lighter

4. In the system shown below consisting of two blocks of mass m connected by a chord, the coefficient of kinetic friction is 0.1 and the coefficient of static friction is 0.2. The pulley and chord are ideal (i.e. frictionless, massless, etc.). Sand can be added to the top block as desired.



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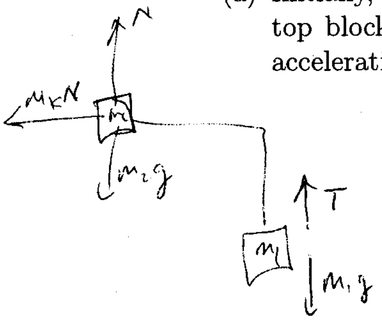
(a) Initially, the system is accelerating to the right, and there is no sand on the top block. What mass of sand must be placed on the top block to bring the acceleration to exactly zero?



~~accel~~ $m_1 a = m_1 g - T$

$m_2 a = T - \mu_K m_2 g$

$(m_1 + m_2) a = (m_1 - \mu_K m_2) g = 0$

$m_2 = \frac{m_1}{\mu_K} = 10m$

∴ add $9m$ of sand

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(b) An additional $1m$ of sand is now added to the top block so the system comes to rest. What mass of sand must then be removed to start the blocks moving again?

$T = m_1 g = \mu_s m_2 g$

$m_2 = \frac{m_1}{\mu_s} = \frac{m}{0.2} = 5m$

~~∴ remove~~

began w/ ~~10m~~ 11m, so remove $6m$ to begin moving.