Exam 1

1. Suppose you are given that the total force on a pointlike object of mass m is given by

$$\vec{F}_{tot} = \hat{x}\,\alpha t + \hat{y}\,\beta$$

where α and β are constants and t is time.

a) Suppose $\vec{v} = 0$ at time t=0. Find the velocity as a function of time t>0.

ANS

$$\vec{F} = m\vec{a}$$

$$\hat{x}\,\alpha t + \hat{y}\,\beta = m\frac{d\vec{v}}{dt}$$

$$\boxed{\frac{1}{m}\left(\hat{x}\,\frac{1}{2}\alpha t^2 + \hat{y}\,\beta t\right) = \vec{v}}$$

b) Suppose the position is at $x=0$ and $y=0$ at time $t=0$ What is the position as a function of time for t>0?

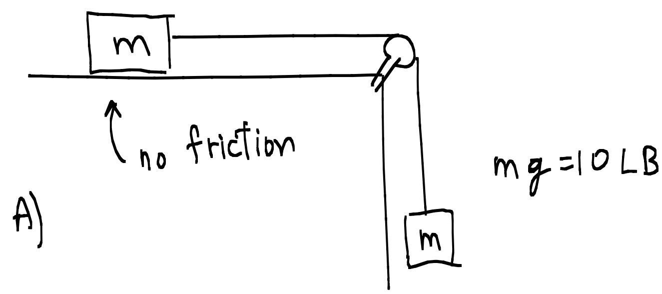
ANS

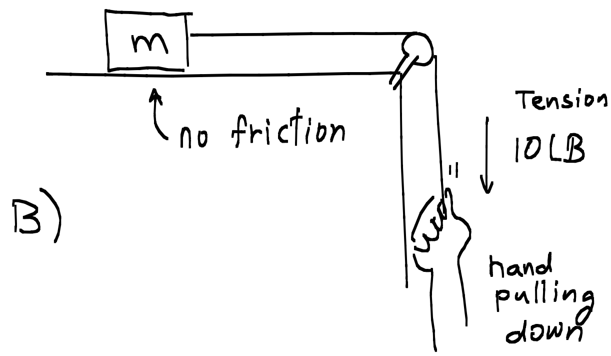
$$m\frac{dx}{dt} = \frac{1}{2}\alpha t^2 \qquad m\frac{dy}{dt} = \beta t$$

$$\boxed{x = \frac{1}{6}\alpha t^3 \frac{1}{m} \qquad y = \frac{1}{2}\beta t^2 \frac{1}{m}}$$

2. Multiple choice questions

i) Consider the two pictures below.



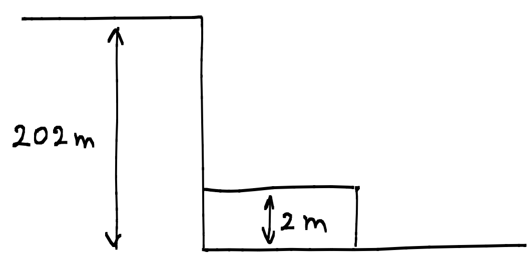


a) The acceleration is greater in case A)
b) The acceleration is greater in case B)
c) The acceleration is the same in both

ANS : b because of less inertia

ii) Suppose a person jumps off a building 202 m high onto cushions having a total thickness of 2 m. If the cushions are crushed to a thickness of 0.5 m, what is the person's average acceleration as the person slows down, assuming that the acceleration is uniform during slowing.

a) g
b) 133 g
c) 5 g
d) 2 g
e) 266 g



ANS

$$2a\Delta x = -v_i^2$$

$$\Delta x = -1.5\text{ m}$$

$$a = \frac{-v_i^2}{2\Delta x} = \frac{v_i^2}{3\text{ m}} \qquad v_i^2 = 2g(200\text{ m})$$

$$\Rightarrow \quad a = \frac{400}{3}g \approx \boxed{133\,g} \qquad \text{b)}$$

iii) What is $\frac{d\hat{\theta}}{dt}$ in polar coordinates?

a) $-\dot{\theta}\,\hat{\theta}$
b) $-\dot{\theta}^2\,\hat{r}$
c) $-\dot{\theta}\,\hat{r}$
d) $\dot{\theta}\,\hat{\theta}$

ANS

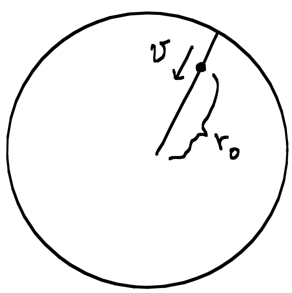
$$\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\cos\theta\,\hat{x} + -\dot{\theta}\sin\theta\,\hat{y} = \boxed{-\dot{\theta}\,\hat{r}} \qquad \text{c)}$$

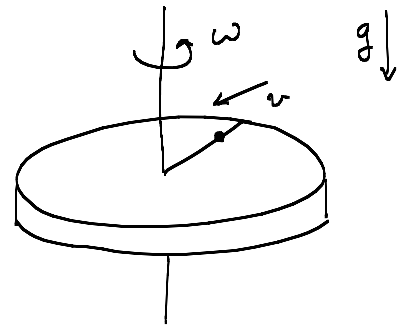
3. An ant of mass M is walking along a radial line painted on a horizontal disk that is rotating with constant angular velocity ω. The ant is moving toward the center of the disk with constant radial speed v along the painted line, having started at $r = r_0$ at time $t = 0$. You may assume that gravity is directed downward and that the friction is large enough that the ant never slips. Find the magnitude of the force on the ant at a time t>0 before it reaches the center of the disk.

(Hint: $\frac{d\hat{r}}{dt} = \omega \hat{\theta}$)

Top view



Side view



ANS

$$\vec{r} = r\,\hat{r} \qquad r = r_0 - vt$$

$$= (r_0 - vt)\,\hat{r} \qquad \hat{r} = \hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)$$

$$\frac{d\vec{r}}{dt} = -v\,\hat{r} + (r_0 - vt)\frac{d\hat{r}}{dt} \qquad \frac{d^2\hat{r}}{dt^2} = -\omega^2\,\hat{r}$$

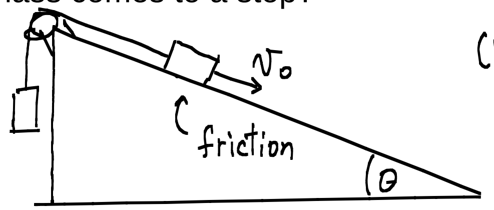
$$\frac{d^2\vec{r}}{dt^2} = -v\frac{d\hat{r}}{dt} - v\frac{d\hat{r}}{dt} + (r_0 - vt)\frac{d^2\hat{r}}{dt^2}$$

$$= -v\omega\hat{\theta}\,2 + (-\omega^2\hat{r})(r_0 - vt)$$

$$= -2\omega v\,\hat{\theta} - \omega^2(r_0 - vt)\,\hat{r}$$

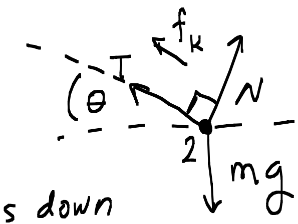
$$|\vec{F}| = M\left|\frac{d^2\vec{r}}{dt^2}\right| = \boxed{M\sqrt{4\omega^2 v^2 + \omega^4 (r_0 - vt)^2}}$$

4. A block is moving at speed v_0 down a hill that makes an angle θ with the horizontal. The coefficient of static friction between the block and the inclined plane is μ_k, and an identical block is attached to the one on the inclined plane through a string draped over a pulley as shown. How far must the block move on the inclined plane before the mass comes to a stop?



(Hint: Define a mass m for the blocks. In the end m drops out.)

ANS



define "+" direction as down or up the incline.

$$ma = mg - T$$

$$f_k = \overbrace{mg\cos\theta}^{N}\,\mu_k$$

$$T + f_k - mg\sin\theta = ma$$

$$\Rightarrow \cancel{m}g - \cancel{m}a + \mu_k \cancel{m}g\cos\theta - \cancel{m}g\sin\theta = \cancel{m}a$$

$$2a = g(1 + \mu_k\cos\theta - \sin\theta)$$

$$a = \frac{g}{2}(1 + \mu_k\cos\theta - \sin\theta)$$

$$-v_0^2 = 2a\Delta x$$

$$\Delta x = \frac{-v_0^2}{2a} = \boxed{\frac{-v_0^2}{g(1 + \mu_k\cos\theta - \sin\theta)}}$$