Name Solutions

Exam #2

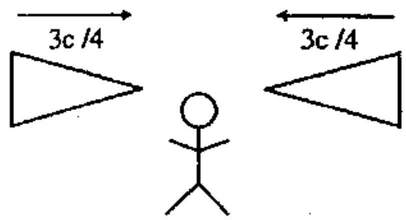
Physics 247

November 8, 2000

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. Two spaceships, each of proper length 100 m, travel toward each other at speeds of $3c/4$ relative to us, as shown.



(a) How long is each ship in our frame?

$$\gamma = \frac{1}{\sqrt{1-9/16}} = \frac{4}{\sqrt{7}}$$

$$L = \frac{L_0}{\gamma} = \frac{\sqrt{7}}{4} \times 100\,m$$

(b) How fast is each ship moving according to the other?

$$\frac{u+v}{1+\frac{uv}{c^2}} = \frac{3/2}{1+\frac{9}{16}} = \frac{3/2 \cdot 16}{25}\,c = \frac{24}{25}\,c$$

(c) How long is each ship according to the other?

$$\gamma = \frac{1}{\sqrt{1-\frac{24^2}{25^2}}} = \frac{25}{7}$$

$$L = 28\,m$$

2. Two rocket ships are moving in the same direction at a speed of $\sqrt{3}c/2$, a distance of 6 light-sec apart, with respect to a stationary observer. The rear ship carries a clock that is synchronized with the observer's clock at $t = t' = 0$. At $t = 0$, the rear ship sends out a light pulse.

(a) At what time on the rear ship's clock does the light pulse reach the forward ship?

~~Rest~~ $\gamma = \frac{1}{\sqrt{1-3/4}} = 2$

$\therefore$ 12 light-sec apart in moving frame

$\therefore$ 12 sec

(b) At what time on the observer's clock does the light pulse reach the forward ship?

$$ct = L + vt$$

$$t = \frac{L}{c-v} = \frac{6\,\text{s}}{1-\sqrt{3}/2}$$

event method (12, 12)

$$t' = \gamma\left(12 + \sqrt{3}/2 \cdot 12\right) = \frac{6\left(1+\frac{\sqrt{3}}{2}\right)}{\left(1+\sqrt{3}/2\right)\left(1-\sqrt{3}/2\right)} = \frac{6}{1-\sqrt{3}/2}$$

3. A particle of mass m is subject to the potential $U(x) = U_0 \left(\frac{x}{a}\right)^4$

(a) Plot U from x = -2a to x = 2a.

(b) Is $x = 0$ a stable, unstable, or neutrally stable equilibrium point?

stable

(c) Is the force on the particle conservative or non-conservative? Why?

cons, comes from a pot

(d) Write an equation for the force on the particle as a function of x.

$$F = -\frac{dU}{dx} = \frac{-4U_0 x^3}{a^4}$$

(e) If the particle starts at rest at the location $x = a/2$, how much work has the force done on the particle when the particle gets to $x = 0$?

$$W = \Delta E = U_0 \left(\frac{a}{2a}\right)^4 = \frac{U_0}{16}$$

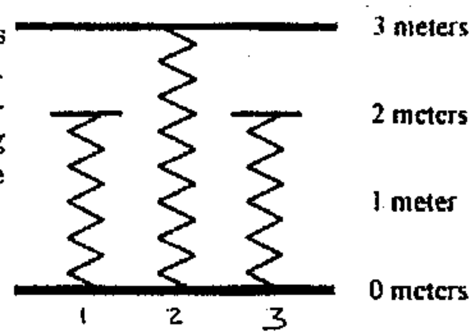
(f) What is its velocity the 17^{th} time it passes $x = 0$?

$$\frac{1}{2}mv^2 = \frac{U_0}{16} \Rightarrow v = -\sqrt{\frac{U_0}{8m}}$$

(g) What is the minimum x-coordinate for the motion?

$$x = -\frac{a}{2}$$

4. The diagram shows a system of three massless springs, each with spring constant $k = 20$ N/m. The middle spring is 3 m long when it is unstretched, and the two side springs are 2 m long when unstretched. Let $z = 0$ at the bottom of the diagram, and take $g = 10$ m/s^2.



(a) Write formulas for the potential energy of the springs as a function of the position z of the top platform. Be sure to write formulas for each of the three individual springs. Note that the top platform will compress the side springs when it is pushed low enough.

$U_1 = \frac{1}{2}k\,(x - 2m)^2 \quad x < 2m \quad$ otherwise 0

$U_2 = \frac{1}{2}k\,(x - 3m)^2 \quad x < 3m \quad$ otherwise 0

$U_3 = \frac{1}{2}k\,(x - 2m)^2 \quad x < 2m \quad$ otherwise 0

(b) A 1 kg block is placed on the platform. You press down on the platform until it is at a height $z = 1$ m. When you release the platform, what is the maximum height the block attains?

$U = \frac{1}{2}k(1m)^2 + \frac{1}{2}k(1)^2 + \frac{k}{2}(2m)^2 = 20 + 40 = 60\,J$

$h = 1m + \frac{U}{mg} = 7m$

(c) If the block weighed 1/10 as much, would it fly higher or lower under the same initial conditions? How much higher or lower?

61m