Name Key

Exam #2

Physics 247

November 5, 2003

Each problem is worth 25 points

Problem	Score
1	25
2	25
3	25
4	25
Total	100

Average 76 raw
86 curved

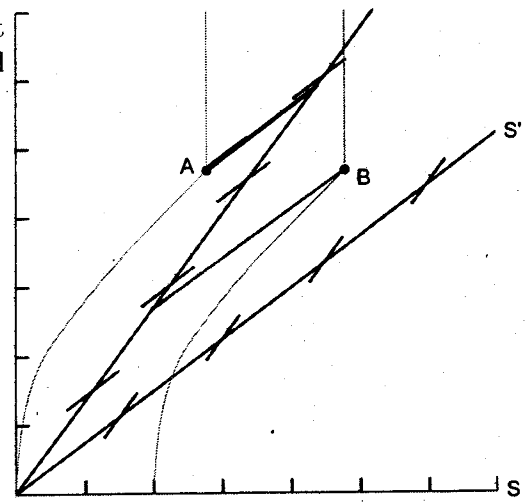
curve $= (\text{raw} - 49)\frac{30}{32} + 60$

1. Two objects have parallel world lines in reference frame S, as shown. Answer the following questions using the Minkowski diagrams. Approximate answers will suffice. Show your measurements on the diagrams. Units are seconds and light-seconds.

(a) According to an observer in S', at what times do the objects reach points A and B?

B - 1.8s

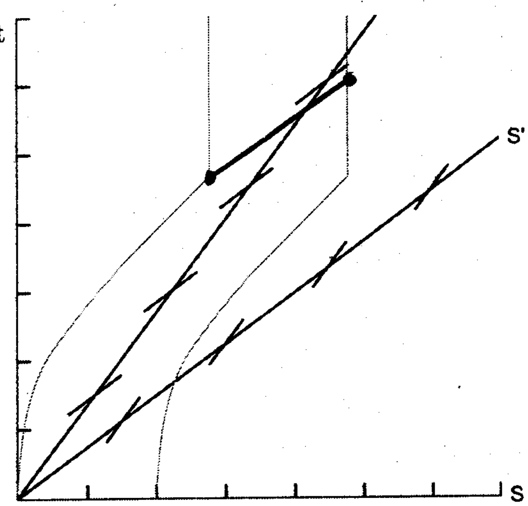
A - 3.9s



(b) Once both objects have stopped in S, what is the distance between them in S'?

$L' = 0.9'' \times \frac{1\,ls}{0.65''}$

$\approx 1.4\,ls$



(c) What is the value of the invariant space-time interval between A and B?

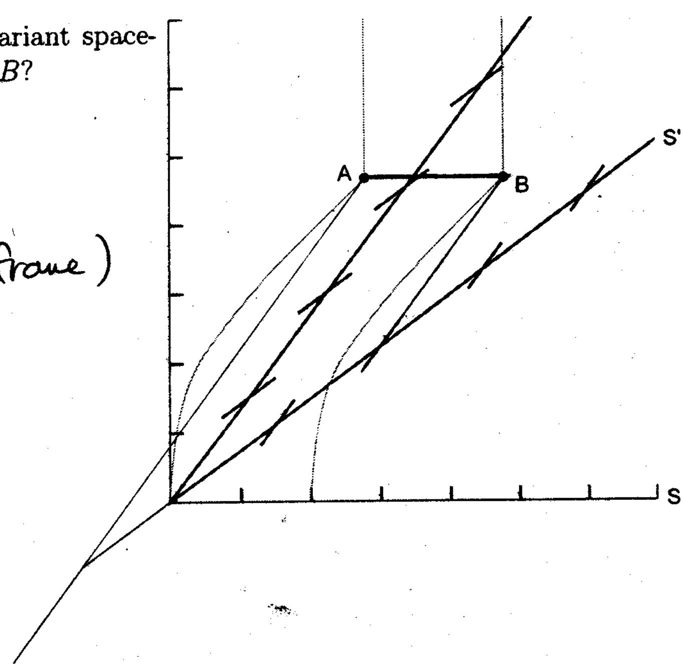
$|\Delta s| = 2\,ls$

(easiest to use S frame)

S' frame measurements

$\Delta s^2 = (2.1\,ls)^2 - (3\,ls)^2$

$= -(2.14\,ls)^2$



2. A particle moves with speed $s = 3c/5$ along the $\hat{y}$-axis in the laboratory.

(a) Find its speed and direction as viewed by an observer moving at $-4c/5$ along the $\hat{x}$-axis.

$$\gamma = \frac{1}{\sqrt{1-16/25}} = 5/3$$

$$v_y' = \frac{v_y}{\gamma\left(1-\frac{v v_x}{c^2}\right)} = \frac{3/5\,c}{5/3\,(1-0)} = \frac{9}{25}c$$

$$v_x' = \frac{v_x - v}{1-\frac{v v_x}{c^2}} = 4/5\,c$$

$$v = \sqrt{\left(\frac{9}{25}\right)^2 + \left(\frac{4}{5}\right)^2}\,c$$

$$\tan\theta = \frac{9/25}{4/5} = \frac{9}{20}$$

(b) Repeat for the observer moving at $-3c/5$ along the $\hat{y}$-axis.

$$v_y' = \frac{v_y - v}{1-\frac{v v_y}{c^2}} = \frac{3/5\,c - (-3/5\,c)}{1-(-3/5)(3/5)} = \frac{6/5\,c}{1+\frac{9}{25}}$$

$$= \frac{30}{34}c$$

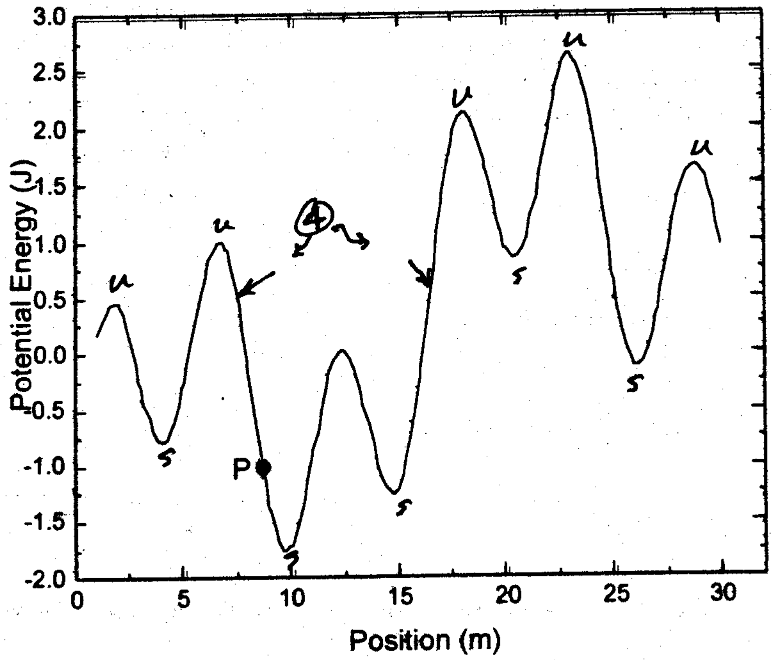
$$v_x' = 0$$

$$v = 30/34\,c$$

$$\tan\theta = \infty$$

3. (a) On the potential energy curve below, plot all of the equilibrium points and label them as either stable (S), unstable (U), or neutral (N).

8



(b) In a system with potential energy given by the curve below, a particle has initial position 12.5 meters and initial kinetic energy 0.5 Joules and is moving to the left. Does this particle experience any turning points? If so, label them carefully with arrows on the plot.

9

Yes

5

(c) If the mass of the particle is 1 kg, what is the speed of this particle at position P?

8

$$\frac{1}{2} m v^2 = 1.5 \text{ J}$$

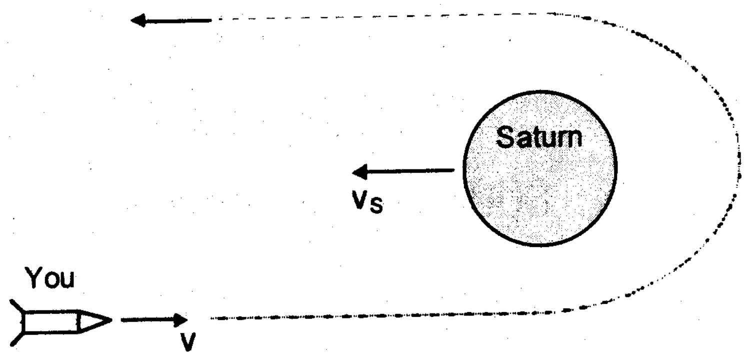
$$v^2 = \frac{3 \text{ J}}{1 \text{ kg}}$$

$$v = \sqrt{3} \ \frac{m}{s}$$

-4 if no $U + K$.

-2 if wrong K.

4. Your spacecraft is drifting without power with velocity $v\hat{x}$ as it encounters the planet Saturn moving at velocity $-v_S\hat{x}$ as shown. If the geometry of the "collision" is as shown below, you are to determine the final speed of your spacecraft after the collision. All speeds are relative to some reference frame that is fixed on the time scale of this problem (like the sun), and are much less than the speed of light. You may assume that your spacecraft is substantially less massive than Saturn, and that your initial and final velocities are parallel to Saturn's velocity.



(a) (18 pts) Write down conservation of energy and momentum for the collision.

$$\frac{1}{2} m v^2 + \frac{1}{2} M v_s^2 = \frac{1}{2} m v'^2 + \frac{1}{2} M v_s'^2 \quad \leftarrow 9$$

$$m v - M v_s = - m v' - M v_s' \quad \leftarrow 9$$

(b) (7 pts) Solve your equations to find the final speed.

-5 if no progress.

-4 if $v' = -v$

$$m(v^2 - v'^2) = M(v_s'^2 - v_s^2)$$

$$m(v + v') = M(v_s - v_s')$$

Divide; $\quad v - v' = -(v_s' + v_s) \quad -2v_s$

$$v' = v + 2v_s$$