Name Solutions

Exam #2

Physics 247

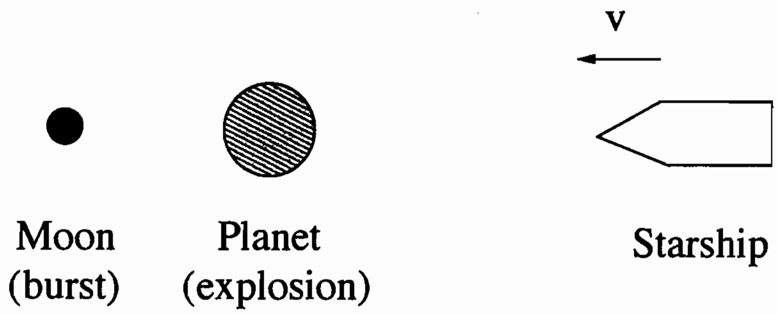
November 2, 2005

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

Useful math: $\sin 2\theta = 2\cos\theta\sin\theta$ | $\cos 2\theta = \cos^2\theta - \sin^2\theta$ | 1 mi=1.609 km

1. An earth starship has been sent to check an Earth outpost on planet P247. The planet's moon houses a battle group of the often hostile Reptulians. As the ship travels toward the planet, it detects a high-energy microwave burst at the Reputlian moon base and then, 10 s later, an explosion at the Earth outpost. The measured space-time coordinates for the two events in the spaceship frame are $s_b = (0, 0)$ and $s_e = (14.9 \text{ light-s}, 14.8 \text{ s})$. The speed of the ship relative to the planet and the base is $0.98c$ ($\gamma = 5$).



(a) As navigational officer, you are asked to find the distance and the time interval between the burst and the explosion as measured in the planet-moon inertial frame.

$x' = \gamma(x - vt)$

$t' = \gamma(t - \frac{v}{c^2} x)$

Unprimed frame is spaceship
Primed frame is Moon-Planet system.

$S_b' = (0,0)$ since $x_b' = 0$ & $t_b' = 0$

$x_e' = \gamma(x_e - vt_e) = 5(14.9 \text{ lt-s} - .98 \cdot 14.8 \text{ lt-s}) = 1.98 \text{ lt-s}$

$t_e' = \gamma(t_e - \frac{v}{c^2} x_e) = 5(14.8s - .98 \cdot 14.9s) = .99s$

$S_e' = (1.98 \text{ lt-s}, .99s)$ $\Rightarrow$ $\Delta dist' = 1.98 \text{ lt-s}$ $\Delta time' = .99s$

(b) Did the Reputlians cause the explosion? Explain.

$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 = (14.8 \text{ lt-s})^2 - (14.9 \text{ lt-s})^2$

$= -2.97 (\text{lt-s})^2 < 0$

$\Rightarrow$ Interval is space like, so two events cannot be causally connected!

2. David Letterman and Jay Leno were both running a marathon. David crossed the finish line at exactly 5pm EST in the New York marathon. Jay on the other hand crossed the finish line at exactly 1/1000 second after 2pm PST (=EST−3 hr) in the LA marathon. Assume that the distance between New York and Los Angeles is 3000 miles.

(a) Do all inertial observers agree on who crossed the finish line first? Why or why not?

Time for David EST = 5pm
Time for Jay EST = 5pm + .001 s
} $\Delta t = .001\,s$

$\Delta x = 3000$ mi $= 4827$ km $= 4.827 \times 10^6$ m

$$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 = 9\times 10^{10}\,m^2 - 2.33\times 10^{13}\,m^2 < 0 \;!$$

Interval is space-like, therefore ordering of events can be different for different observers.

(b) If there exists an observer who concludes that Jay crosses the finish line first, what is the minimum velocity of this observer and in what direction does this observer move (towards or away from New York)?

Minimum ⇒ Cross at same time

$\Rightarrow \Delta t' = 0$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 0$$

$$\Rightarrow \frac{v}{c} = \frac{\Delta t}{\Delta x}\cdot c = \boxed{.062}$$

Towards L.A.

or

Away from New York.

3. A hovercraft of length L travels at velocity $v = 0.8c$ past you. On the craft is a clock. Once a minute the clock triggers a paint gun that fires and thereby leaves a paint mark on the ground just below the hovercraft.

(a) How far apart on the ground are the paint marks (in light-minutes)?

due to time dilation, marks are made every

$$\Delta t = \gamma \tau$$

$$\Delta x = v \Delta t = v \gamma \tau = (0.8c)(5/3) \cdot 1\,\text{min}$$

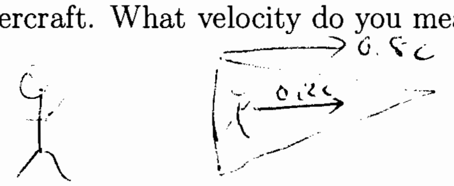
$$= \frac{4}{3}\ \ell\text{-min}$$

$(2.4 \times 10^{10}\,\text{m})$

(b) What is the length of the spacecraft as measured by you?

$$\frac{L_0}{\gamma} = \frac{3}{5} L$$

(c) On the hovercraft, Bobby Jenks throws a baseball at $0.2c$ from the back to the front of the hovercraft. What velocity do you measure?



$$u' = \frac{v + u}{1 + \frac{uv}{c^2}} = 0.86c$$

$$= 2.58 \times 10^{8}\,\text{m/s}$$

4. In 1998 Marco Pantani rode his bicycle up Alpe d'Huez in 37 minutes, 35 seconds. The Alpe is 1120 m high, and the road is 14 km long.

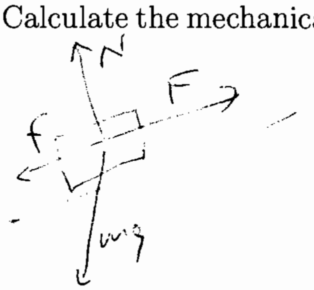
(a) Assuming Pantani's mass (including the bicycle) was 60 kg, what was his average power output? (When riding steep mountains with state-of-the-art bicycles, factors such as wind resistance and friction are small).

$$P = \frac{W}{t} = \frac{mgH}{t} = \frac{(60\,kg)(9.8\,m/s^2)(1120\,m)}{2255\,s}$$

$$= 292\ W$$

(total energy 6.6×10^5 J)

(b) You are pushing a 60 kg object up the Alpe. The coefficient of sliding friction is 0.4. Calculate the mechanical advantage (mg/F, where F is the force you apply to the object).



$$F = f + mg\sin\theta$$

$$= \mu_k mg\cos\theta + mg\sin\theta$$

$$\frac{mg}{F} = \frac{1}{\mu_k\cos\theta + \sin\theta} = 2.1$$

$\sin\theta = \frac{1120}{14000} \Rightarrow \theta = 4.59°$

(c) Calculate the total work done when you reach the top.

$$W = \int \vec{F} d\vec{s} = mg(\mu_k\cos\theta + \sin\theta)L$$

$$= \frac{60\,kg\,(9.8\,m/s^2)}{2.1} \cdot 14000\,m =$$

$$= 3.9 \times 10^6\ J$$

(factor 6 more than Pantani did)