Exam II Solutions

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i) length contraction, $\frac{L}{\gamma} = \frac{100\,m}{5/4} = 80\,m$

ii) no length contraction in transverse directions, $L = 60\,m$

iii) proper time is in the Earth frame, so $\Delta t_{rocket\,frame} = \gamma \Delta t_{Earth\,frame} = (\frac{5}{4})(3\,hrs) \approx 3.8\,hrs$

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i) yes. as long as the slope of the worldline connecting the two events (team A buzzing and team B buzzing) is less then 1, you can always find a frame whose velocity relative to the "host's frame" is less than c, but greater than the velocity for the two events to be simultaneous (which is found in part ii)). in this primed frame, team B buzzing occurs before team A buzzes.

ii) yes, when, in the primed frame, $\Delta t' = 0$

$\gamma(\Delta t - \frac{v}{c^2}\Delta x) = 0$

$\Delta t = \frac{v}{c^2}\Delta x$

$\frac{c\Delta t}{\Delta x} = \frac{v}{c} = \beta$. however, $\frac{c\Delta t}{\Delta x}$ is the slope of the worldline (in the unprimed "host's" frame) connecting team A and team B buzzing, which is $\frac{1}{2}$.

so, $v = \frac{1}{2}c$

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i) $\Delta t_{trip} = \frac{26\,ly}{.99c} = 26.3 years$

ii) $\Delta t_{signal} = \frac{26}{.99c} years + 26 years = 52.3 years$

iii) $\Delta t_{travler} = \frac{\Delta t_{trip}}{\gamma} = \frac{26\,ly}{.99c}\frac{1}{\gamma} = 3.71 years$, where $\gamma = 7.09$.

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i) if gas is moving towards us, then the light should be of higher frequency and lower wavelength - blue shifted. this corresponds to the $\lambda = 499.8nm$ peak curve.

ii) to find the velocity, we need to use both the blue shifted and red shifted peaks with the Doppler shift equations for each.

blue shifted, moving towards us: $f_{blue} = \frac{c}{\lambda = 499.8\,nm} = f_0\sqrt{\frac{1+\beta}{1-\beta}} \rightarrow \frac{c}{\lambda = 499.8\,nm}\sqrt{\frac{1-\beta}{1+\beta}} = f_0$

red shifted, moving away from us: $f_{red} = \frac{c}{\lambda = 501.6\,nm} = f_0\sqrt{\frac{1-\beta}{1+\beta}} \rightarrow \frac{c}{\lambda = 501.6\,nm}\sqrt{\frac{1+\beta}{1-\beta}} = f_0$

setting the two equations equal to each other

$\frac{c}{\lambda = 499.8\,nm}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{c}{\lambda = 501.6\,nm}\sqrt{\frac{1+\beta}{1-\beta}}$

remember, we are looking to solve for β.

so, $\frac{501.6\,nm}{499.8\,nm} = \frac{1+\beta}{1-\beta}$

solving for β , we see that $\beta = 0.00180$, or $v = 0.00180c$.

iii) the gravitational force serves as the centripital force to keep the gas in circular motion:

$\frac{Gm_{gas}M_{core}}{r^2} = \frac{m_{gas}v^2}{r}$

solving for M_{core}

$M_{core} = \frac{v^2 r}{G} = 4.1 \times 10^{39}\,kg$, where $v = 0.00180c$ from part ii), $r = 100\,ly$ as stated in the problem, and G is the gravitational constant