1. A rocket of initial mass 10 kg burns until its mass is 1 kg. The rocket starts at rest, and its ejected gas leaves at a velocity of 2000 m/s.

   (a) If the rocket moves in a region of space with no gravitational field, what is its velocity at the end of the burn?

   (b) After the burn, the rocket collides inelasitically with a piece of debris which is at rest. The mass of the debris is 1kg, the collision is head on, and the rocket and debris stick together after the collision. What is their velocity after the collision?

   (c) If instead the rocket collides elastically with the debris, what is the <u>rocket</u>'s velocity after the collision? Assume the debris moves in the direction of the rocket's initial velocity.

a) $2000\,\text{m/s} \ln 10 = v_0$

b) $1000\,\text{m/s} \ln 10 = \frac{v_0}{2}$

c) Same mass, cons. of momentum and energy required $v_{\text{rocket}} = 0$, $v_{\text{deb}} = v_0$

2. A 0.4 kg piece of putty moves horizontally at $v = 10$ m/s towards a meter stick. The meter stick hangs from a pivot point at one end, and the mass of the meter stick is 1.2 kg. The putty strikes the meter stick at the 50 cm mark and sticks. The moment of inertia about the center of mass of a rod of mass M and length L is $ML^2/12$.

   (a) What is the angular velocity of the meter stick/putty system just after impact?

   (b) Does the meter stick putty system rotate through 360 degrees around the pivot after the collision? Explain. Let g=10 m/s^2.

$$L = \frac{mvl}{2} = I\omega$$

$$\omega = \frac{mvl}{2\left(\underbrace{\frac{ML^2}{12} + \frac{ML^2}{4}}_{ML^2/3} + \frac{mL^2}{4}\right)} = \frac{0.4\,\text{kg}\,(10\,\text{m/s})(0.5\,\text{m})}{\underbrace{\left(\frac{1.2\,\text{kg}\cdot 1\,\text{m}^2}{3} + \frac{0.4\,\text{kg}\ 1\,\text{m}^2}{4}\right)}_{0.5\,\text{kgm}^2}}$$

$$= 4\,\text{rad/s}$$

b) $E = \frac{1}{2} I\omega^2 = \frac{1}{2}(0.5\,\text{kgm}^2)\left(\frac{16}{s^2}\right) = 4\,\text{J}$

$(m+M)gh = 1.6\,\text{kg} \cdot \frac{10\,\text{m}}{s^2} \cdot 1\,\text{m} = 16\,\text{J}$

doesn't go over top

3. A moving proton collides head-on with a photon (ϕ). a) Find the invariant mass of the pair of particles. You should write your answer in terms of photon's energy E_ϕ and the proton's mass m_p and total energy E_p. b) Taking $E_p \gg m_p c^2$, i.e. $\sqrt{E_p^2 - m_p^2 c^4} \approx E_p$, find the threshold proton energy for the reaction

$$p + \phi \longrightarrow p + e^+ + e^-$$

The electron mass is m_e.

$$(E_p + E_\phi)^2 - (p_p - p_\phi)^2 c^2 = M_{inv}^2 c^4$$

$$\underbrace{E_p^2 + \cancel{E_\phi^2} + 2E_p E_\phi - p_p^2 c^2}_{m_p^2 c^4} + 2 p_p p_\phi c^2 - \cancel{p_\phi^2 c^2} =$$

$$= m_p^2 c^4 + 2E_\phi (E_p + p_p c) = M_{inv}^2 c^4$$

a)

$$m_p^2 c^4 + 2E_\phi \Big(\underbrace{E_p + \sqrt{E_p^2 - m_p^2 c^4}}_{\approx 2E_p}\Big) = M_{inv}^2 c^4$$

b) $m_p^2 c^4 + 4 E_\phi E_p = (m_p + 2m_e)^2 c^4$

$$E_p = \frac{(4m_e^2 + 4m_e m_p) c^4}{4 E_\phi}$$

$$= \frac{m_e (m_e + m_p) c^4}{E_\phi}$$

4. Two identical (relativistic) particles of mass m each have kinetic energy T in the center of mass frame. Find each of their kinetic energies in a frame where one of them is at rest. Hint: invariant mass is the easiest way to work this problem.

$$(E_1 + mc^2)^2 - p_1^2 = [2(T+m)]^2$$

$$2\underbrace{E_1}_{T_1+m} mc^2 + m^2c^4 + m^2c^4 = 4T^2 + 8Tm + 4m^2$$

$$mc^2 2T_1 = 8Tmc^2 + 4T^2$$

$$T_1 = 4T + \frac{2T^2}{mc^2}$$