Name Solutions

Exam #3

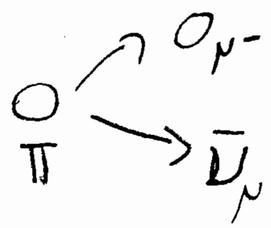
Physics 247

December 7, 2005

Each problem is worth 25 points

Problem	Score
1	
2	
3	
4	
Total	

1. A pion at rest decays to a muon and an antineutrino. Find the kinetic energy of the muon and the energy of the antineutrino. The masses are $m_\pi = 273 m_e$, $m_{mu} = 207 m_e$ where $m_e = 0.51$ MeV and we can assume that the antineutrino is massless for this problem.



$$m_\pi c^2 = \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} + p_\nu c \qquad \text{Energy}$$

$$p_\nu = p_\mu \qquad \text{Momentum}$$

$$\Rightarrow (m_\pi c^2 - p_\nu c)^2 = p_\nu^2 c^2 + m_\mu^2 c^4$$

$$m_\pi^2 c^4 - 2 p_\nu m_\pi c^3 + \cancel{p_\nu^2 c^2} = \cancel{p_\nu^2 c^2} + m_\mu^2 c^4$$

$$\Rightarrow \quad p_\nu = \frac{(m_\pi^2 - m_\mu^2) c^4}{2 m_\pi c^3} = \boxed{29.8 \text{ MeV}/c} \qquad 29.59$$

$$\Rightarrow E_\nu = p_\nu c = \boxed{29.8 \text{ MeV}} \qquad 29.59$$

$$E_\mu = \sqrt{m_\mu^2 c^4 + p_\mu^2 c^2} = 109.8 \text{ MeV} \qquad 109.64$$

$$\Rightarrow \quad K.E._\mu = E_\mu - m_\mu c^2$$

$$= \boxed{4.1 \text{ MeV}}$$

2. Particle physicists discover new elementary particles using high energy collisions. They use either fixed targets (part a) or colliding beams (part b).

(a) In a fixed target experiment a proton with kinetic energy K collides with a proton at rest. If a new particle q with mass M is created via the reaction $p + p \longrightarrow p + p + q$, what is M in terms of K and m_p?

Use invariant mass:

In fixed target frame: $m_{inv}^2 c^4 = (E_p + m_p c^2)^2 - p_p^2$

In center of momentum frame after collision: $m_{inv}^2 c^4 = (2m_p + M)^2 c^4$

$\Rightarrow (E_p + m_p c^2)^2 - p_p^2 = (2m_p + M)^2 c^4 \Rightarrow E_p^2 - p_p^2 + 2E_p m_p c^2 + m_p c^2 = (2m_p + M)^2 c^4$

$\Rightarrow 2((K + m_p c^2) m_p c^2 + m_p^2 c^4) = 2m_p c^2 K + 4 m_p^2 c^4 = (2m_p + M)^2$

$\Rightarrow \boxed{Mc^2 = \sqrt{2m_p c^2 (K + 2m_p c^2)} - 2m_p c^2}$

(b) In a colliding beam experiment the protons are moving towards each other with equal and opposite velocity. Let K be the total kinetic energy of the protons. What is M in terms of K and m_p? Total momentum is zero.

Initial energy $= K + 2m_p$

Final Energy $= 2m_p + M \leftarrow$ All at rest

$\Rightarrow \boxed{M = K}$

(c) Typically the kinetic energy K involved is much bigger than the rest mass energy $m_p c^2$. If you were to build an experiment to discover new massive particles, would you rather have a fixed target collider or one with colliding beams?

Would want a colliding beam accelerator, because mass of discoverable new particle increases faster with larger kinetic energy, i.e. K is larger than $\sqrt{m_p c^2 K}$.

3. Two objects of masses m_1 and m_2 have equal and opposite velocities v. They collide elastically, leaving m_1 at rest.

(a) What is the mass ratio m_2/m_1?

Elastic collision: $V_{1i} - V_{2i} = V_{2f} - V_{1f} \Rightarrow 2V = V_{2f} - 0 \Rightarrow \boxed{V_{2f} = 2V}$

$(m_1 - m_2)\cancel{v} = 2m_2\cancel{v}$ by momentum conservation

$\Rightarrow \boxed{\frac{m_2}{m_1} = \frac{1}{3}}$

(b) What is the velocity of m_2 after the collision?

$\boxed{V_{2f} = 2V}$

(c) If the collision is vertical, how high will m_2 go?

$\frac{1}{2}\cancel{m_2} V_{2f}^2 = \cancel{m_2} g h$ $\boxed{h = \frac{2v^2}{g}}$

Energy Conservation

(d) Immediately after the collision, m_1 is held still and then released just as m_2 returns for a second collision. What is the velocity of m_2 after this collision?

Exactly the reverse situation

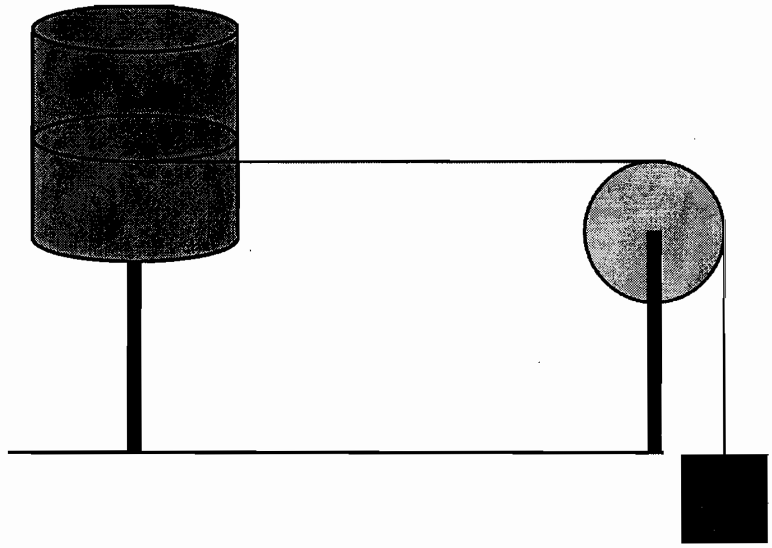
$\Rightarrow \boxed{V_{2f} = V}$

or $2V = V_{2f} - V_{1f} \Rightarrow V_{1f} = V_{2f} - 2V$, $m_1 = 3m_2$

$-m_2(2V) = m_1 V_{1f} + m_2 V_{2f} \Rightarrow -2m_2 V = m_1 V_{2f} - 2m_1 V + m_2 V_{2f}$

$\Rightarrow V_{2f} = \frac{2(m_1 - m_2)V}{m_1 + m_2} = \frac{2(2)}{4} V = \boxed{V}$

4. A cylinder of radius R and moment of intertia I_c rotates about a vertical axis without friction. A cord, wrapped around the cylinder, is attached to a hanging mass m via a pulley (moment of inertia I_p, radius r). The mass is dropped a distance h, following which the cord drops off the cylinder.



(a) What is the angular rotation frequency of the cylinder?

Use Energy Conservation

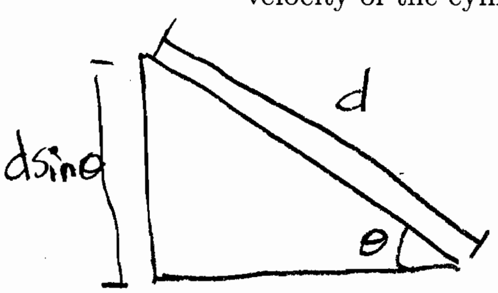
$$\omega_p = \frac{v}{r} \qquad \omega_c = \frac{v}{R}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_p\omega_p^2 + \frac{1}{2}I_c\omega_c^2$$

$$= \left(\frac{1}{2}m + \frac{1}{2}\frac{I_p}{r^2} + \frac{1}{2}\frac{I_c}{R^2}\right)v^2 \Rightarrow v^2 = \frac{2mgh}{m + \frac{I_p}{r^2} + \frac{I_c}{R^2}}$$

$$\boxed{\omega_c^2 = \frac{2mgh}{R^2\left(m + \frac{I_p}{r^2} + \frac{I_c}{R^2}\right)}}$$

(b) The spinning cylinder is now placed at the top of a ramp tilted an angle θ from the horizontal. The cylinder rolls with slipping down the ramp a distance d. What is the velocity of the cylinder?



$$E_i = M_c g d\sin\theta + \frac{1}{2}I_c\omega_c^2$$

$$E_f = \frac{1}{2}M_c v^2 + \frac{1}{2}I_c\omega_f^2 = \frac{1}{2}\left(M_c + \frac{I_c}{R^2}\right)v^2$$

$$\boxed{v = \left[\frac{\left(2M_c g d\sin\theta + \frac{4 I_c m^2 g^2 h^2}{R^2\left(m + \frac{I_p}{r^2} + \frac{I_c}{R^2}\right)^2}\right)}{M_c + \frac{I_c}{R^2}}\right]^{1/2}} = \left(\frac{2M_c g d\sin\theta + \omega_c^2 I_c}{M_c + I_c/R^2}\right)^{1/2}$$

$$\frac{1}{2} m \underbrace{v_i}_{\omega_c R}^2 + \frac{1}{2} I_c \omega_c^2 + mgh = \frac{1}{2} m v_f^2 + \frac{1}{2} I_c \underbrace{\omega_f}_{v/R}^2$$

$$mgh + \frac{1}{2} I_c \omega_c^2 + \frac{1}{2} m \omega_c^2 R^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I_c \frac{v_f^2}{R^2}$$

$$v_f^2 = \frac{mgh + \frac{1}{2} I_c \omega_c^2 + \frac{1}{2} m \omega_c^2 R^2}{\frac{1}{2} m R^2}$$

$$\frac{1}{2} I \omega_c^2 = \frac{1}{2} m v^2 + \frac{1}{2} I \underbrace{\omega^2}_{\frac{v^2}{R^2}}$$

$$= \frac{1}{2} \left(m + I/R^2\right) v^2$$