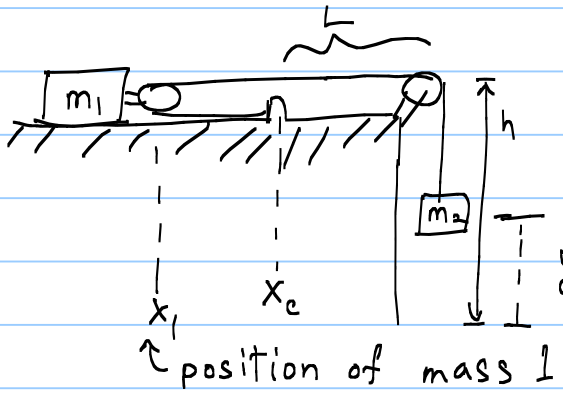
Homework 3

Note Title 9/22/2006

(1) TM 4-83



$m_1 = 20$ kg
$m_2 = 5$ kg

a) $2(x_c - x_1) + L + h - y$ = length of string = constant

$\therefore \quad -2\Delta x_1 - \Delta y = 0$

If $\quad \Delta y = -10\,cm \Longrightarrow \boxed{\Delta x_1 = 5\,cm}$

b) $-m_2 g + T = m_2 \ddot{y} \qquad (*)$

$2T = m_1 \ddot{x}_1$

Since $\quad -2\ddot{x}_1 - \ddot{y} = 0$ from part a),

we have

$$T = \frac{m_1 \ddot{x}_1}{2} = \frac{m_1}{2}\left(\frac{-\ddot{y}}{2}\right)$$

Plugging this into $(*)$, we find

$$-m_2 g + -\frac{m_1 \ddot{y}}{4} = m_2 \ddot{y}$$

$$\therefore \quad \ddot{y}\left(\frac{m_1}{4} + m_2\right) = -m_2 g$$

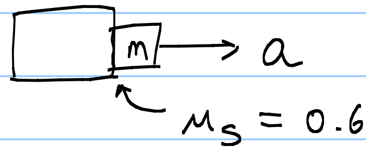
$$\boxed{\ddot{y} = \frac{-m_2 g}{\left(\frac{m_1}{4} + m_2\right)}} \approx \boxed{4.9 \frac{m}{s^2}}$$

$$\therefore \quad \boxed{T = -\frac{m_1 \ddot{y}}{4} = \frac{m_1 m_2 g}{(m_1 + 4m_2)}} \approx \boxed{24.5\,N}$$

$$\therefore \quad \boxed{\ddot{x}_1 = \frac{-\ddot{y}}{2} \approx -2.45 \frac{m}{s^2}}$$

$$\boxed{\ddot{y}_1 \approx 4.9 \frac{m}{s^2}}$$

② $\boxed{\text{TM } 5-42}$



$m = 2\,kg$

a) min acc.

$$ma_{min} = F_{N_{min}} \qquad mg = f_{s_{max}} = \mu_s F_{N_{min}}$$

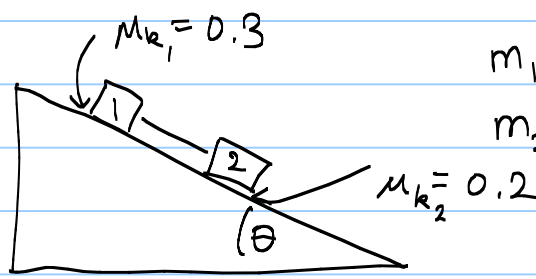
$$\Rightarrow \boxed{a_{min} = \frac{g}{\mu_s}} \approx \frac{9.8\ \frac{m}{s^2}}{0.6} \approx \boxed{16\ \frac{m}{s^2}}$$

b) $f_s = mg \approx \boxed{19.6\ N}$

c) f_s remains the same even if $a > a_{min}$

d) shown in part a)

③ $\boxed{\text{TM } 5-43}$

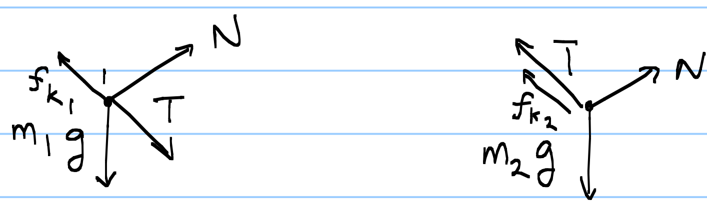


$m_1 = 0.8\ kg$

$m_2 = 0.25\,kg$

$\theta = 10°$

a) Find $\vec{a}$ for blocks



Let s_1 be the 1 displacement parallel to the incline.

" s_2 " " 2 " " " " "

$$m_1 \ddot{s}_1 = \left[T - f_{k_1} + m_1 g \sin\theta\right]$$
$$m_2 \ddot{s}_2 = \left[-T - f_{k_2} + m_2 g \sin\theta\right]$$

$$f_{k_1} = (m_1 g \cos\theta)\,\mu_{k_1} \qquad f_{k_2} = (m_2 g \cos\theta)\,\mu_{k_2}$$

$\ddot{s}_1 = \ddot{s}_2$ since string is taut

$\therefore$ Summing,

$$(m_1 + m_2)\,\ddot{s}_1 = -(f_{k_1} + f_{k_2}) + (m_1 + m_2) g \sin\theta$$

$$\ddot{s}_1 = \frac{-g\cos\theta\,(m_1\mu_{k_1} + m_2\mu_{k_2})}{m_1 + m_2} + g\sin\theta$$

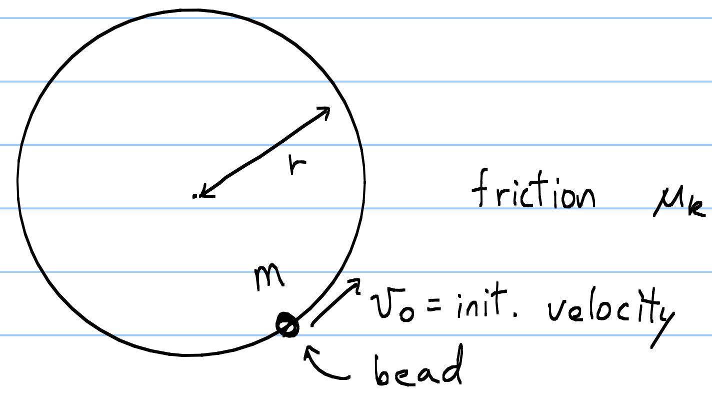
$$= \left(9.8\,\frac{m}{s^2}\right)\left[-\left(\cos\frac{10\pi}{180}\right)\left(\frac{0.8\,kg\;0.3 + 0.25\,kg\;0.2}{0.8\,kg + 0.25\,kg}\right) + \sin\frac{10\pi}{180}\right]$$

$$= \boxed{-0.96\,\frac{m}{s^2}}$$

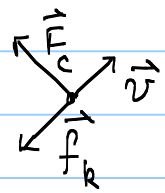
"$-$" = up the incline!

$$T = -m_2\ddot{s}_2 - f_{k_2} + m_2 g \sin\theta$$
$$= m_2\left[-\ddot{s}_2 - g\left(\mu_{k_2}\cos\theta - \sin\theta\right)\right]$$
$$= m_2\left[0.96\,\frac{m}{s^2} - \left(9.8\,\frac{m}{s^2}\right)\left(0.2\cos\frac{10\pi}{180} - \sin\frac{10\pi}{180}\right)\right]$$
$$\approx (0.25\,kg)\left[0.73\,\frac{m}{s^2}\right] \approx \boxed{0.18\ N}$$

④ $\boxed{\text{TM } 5-80}$



Find speed as a function of time.



$$\vec{f}_k = -\hat{v}\,|\vec{F}_c|\,\mu_k$$

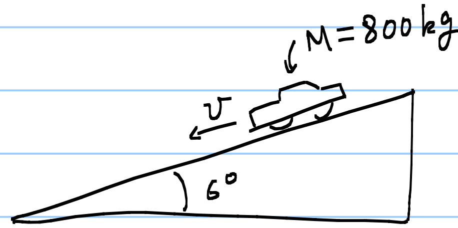
$$\frac{dv}{dt} = -|\vec{f}_k| = -|\vec{F}_c|\,\mu_k$$
$$= -\left(\frac{v^2}{r}\right)\mu_k$$

$$\therefore \int_{v_0}^{v_f} \frac{dv}{v^2} = -\frac{\mu_k}{r}\int_{t_0}^{t_f} dt$$

$$\frac{1}{v_0} - \frac{1}{v_f} = -\frac{\mu_k}{r}\left(t_f - t_0\right)$$

$$\therefore \boxed{v(t) = \left[\frac{1}{v_0} + \frac{\mu_k}{r}\left(t - t_0\right)\right]^{-1}}$$

⑤ $\boxed{\text{TM } 5-95}$



drag: $F_d = 100\,N + \left(1.2\,\frac{N\,s^2}{m^2}\right)v^2$

terminal velocity is reached when

$$F_d = Mg\sin 6^\circ$$
$$\approx (800\,kg)\left(9.8\,\frac{m}{s^2}\right)(0.1) \approx 820\,N$$

$$\therefore \quad v \approx \sqrt{\frac{820\,N - 100\,N}{1.2\ \frac{N s^2}{m^2}}} \approx \boxed{24.5\ \frac{m}{s}}$$

(6) a) $s = L\phi$

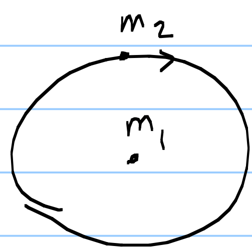
b) $\cancel{m}\ddot{s} = -\cancel{m}g\sin\phi$

$$\boxed{L\ddot{\phi} = -g\sin\phi}$$

c) $$\boxed{\ddot{\phi} = -\frac{g}{L}\phi}$$

d) $$\boxed{\phi = \phi_i \cos\left(\sqrt{\frac{g}{L}}(t - t_i)\right)}$$

(7)



a) $$-G\frac{m_1 \cancel{m_2}}{r^2}\hat{r}(t) = \vec{F} = \cancel{m_2}\frac{d^2\vec{r}}{dt^2}$$

$$\frac{d^2\hat{r}}{dt^2} = \frac{d}{dt}\left[\frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}\right]$$

$$= \cancel{\frac{d^2r}{dt^2}} + 2\cancel{\frac{dr}{dt}}\frac{d\hat{r}}{dt} + r\frac{d^2\hat{r}}{dt^2}$$

0 0 since radius = const.

$$\frac{d^2\hat{r}}{dt^2} = -\frac{Gm_1}{r^3}\hat{r}$$

b) $$\frac{d^2 x_u}{dt^2} = -\frac{Gm_1}{r^3} x_u$$

$$\frac{d^2 y_u}{dt^2} = -\frac{Gm_1}{r^3} y_u$$

$$\dot{x}_u \ddot{x}_u = -\frac{Gm_1}{r^3} x_u \dot{x}_u$$

$$\frac{1}{2}\left(\dot{x}_u^2 - \cancel{\dot{x}_{ui}^2}^{\,0}\right) = -\frac{Gm_1}{2r^3}\left(x_u^2 - \cancel{x_i^2}^{\,r^2}\right)$$

$$\frac{1}{2}\left(\dot{y}_u^2 - \dot{y}_{ui}^2\right) = \frac{-Gm_1}{2r^3}\left(y_u^2 - y_{ui}^2\right)$$

Since $\sqrt{x_{ui}^2 + y_{ui}^2} = r$ and $x_{ui} = r$,

$$\boxed{y_{ui} = 0}$$

$$\therefore \quad \dot{x}_u^2 = -\frac{Gm_1}{r^3}\left(x_u^2 - r^2\right)$$

$$\int \frac{dx_u}{\sqrt{r^2 - x_u^2}} = \sqrt{\frac{Gm_1}{r^3}} \int dt$$

Same as spring case $\Rightarrow$

$$\boxed{x_u = r\cos\left(\sqrt{\frac{Gm_1}{r^3}}\, t\right)}$$

c) $$\boxed{T = 2\pi\sqrt{\frac{r^3}{Gm_1}}}$$ should be familiar from HW 1.