HW # 10 Solutions

$\boxed{2\text{-}7}$ (a) $V = \dfrac{\text{Earth-Moon distance}}{\text{time}} = \dfrac{3.8 \times 10^8 \text{ m}}{1.5 \text{ s}} = 0.84c$ *

(b) $E_K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1-0.84^2}} - 1\right) 938.3 \text{ MeV}$

$= 813 \text{ MeV}$ *

(c) $m(u) = \dfrac{m}{\sqrt{1-u^2/c^2}} = \dfrac{938 \text{ MeV}}{\sqrt{1-0.84^2}} = 1.73 \dfrac{\text{GeV}}{c^2}$ *

(d) Classically $E_K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\left(938 \dfrac{\text{MeV}}{c^2}\right)(0.84c)^2$

$= 331 \text{ MeV}$

% error $= \dfrac{813-331}{813} \times 100\% = 59\%$

$\boxed{2\text{-}9}$ a) $E = \gamma mc^2$

$200 \text{ GeV} = \gamma \, m_{proton} c^2$ where $m_{proton} c^2 = 0.938 \text{ GeV}$

$\Rightarrow \quad \gamma = \dfrac{200 \text{ GeV}}{m_{proton} c^2}$

$$\gamma = \frac{200\,\text{GeV}}{0.938\,\text{GeV}} = 213$$

$$\Rightarrow \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = 213$$

$$\Rightarrow u = 0.999989c \quad \#$$

b) $E \simeq pc$ for $E \gg mc^2$

$$\Rightarrow p = \frac{E}{c^2} = 3.94\times10^4 \frac{\text{GeV}}{c^2}$$

(c) Assume one Au nucleus (system S') to be moving in the $+x$ direction of the lab (system S) then u for the second Au nucleus is in the $-x$ direction. the second Au's energy measured in the S' system is

$$E' = \gamma\,(E + v p_x)$$
$$p' = -\gamma\,(p_x + \frac{v}{c^2}E)$$

$$\Rightarrow E' = 1.68\times10^7\ \text{GeV}$$
$$p'_x = -1.68\times10^7\ \text{GeV}/c$$

$\boxed{2\text{-}21}$ In center of mass frame.

$\bar{p}$ $\rightarrow$ $\quad$ p $\leftarrow$

$P_{total} = 0$

$E_{total} = 2\gamma m_p c^2$

Threshold energy = energy needed to create p, $\bar{p}$ & π^0 at rest

$$2\gamma m_p c^2 = 2 m_p c^2 + m_\pi c^2$$

$$\gamma = \frac{2m_p + m_\pi}{2m_p} = 1 + \frac{m_\pi}{2m_p}$$

The velocity of p in lab $= v$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma = 1 + \frac{m_\pi}{2m_p}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{\left(1+\frac{m_\pi}{2m_p}\right)^2}$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(1+\frac{m_\pi}{2m_p}\right)^2}}$$

$$= \frac{\sqrt{\left(1+\frac{m_\pi}{2m_p}+1\right)\left(1+\frac{m_\pi}{2m_p}-1\right)}}{1+\frac{m_\pi}{2m_p}}$$

$$\frac{v}{c} = \frac{\sqrt{\left(2+\frac{m_\pi}{2m_p}\right)\left(\frac{m_\pi}{2m_p}\right)}}{1+\frac{m_\pi}{2m_p}}$$

$$= \frac{\sqrt{\left(2+\frac{135}{2\cdot(938.272)}\right)\left(\frac{135}{2\cdot(938.272)}\right)}}{1+\frac{135}{2\cdot 938.272}}$$

$$= 0.36$$

The velocity of p in the lab frame

$$u' = \frac{2u}{1+\frac{u^2}{c^2}} = 0.637c$$

$$E_k = (\gamma'-1)m_pc^2 \qquad \text{where } \gamma' = \frac{1}{\sqrt{1-\frac{u'^2}{c^2}}}$$

$$= 280\text{ MeV} \#$$

$$= \frac{1}{\sqrt{1-0.637^2}}$$

$$= 1.297$$

$\boxed{2\text{-}23}$ Conservation of momentum $\Rightarrow$ pions have equal and opposite momentum

$\Rightarrow$ $\pi^{\pm}$ have the same kinetic energy

Conservation of energy

$$\Rightarrow (E_K + m_\pi c^2) \times 2 = m_K c^2$$

$$\Rightarrow E_K = \frac{m_K c^2 - 2 m_\pi c^2}{2}$$

$$= \frac{497.7 - 2 \times 139.6}{2} \text{ MeV}$$

$$= 109.25 \text{ MeV}$$

$\boxed{2\text{-}39}$ a) $E = \gamma m_e c^2 \Rightarrow \gamma = \frac{E}{m_e c^2} = \frac{50\times10^3 \text{ MeV}}{0.511 \text{ MeV}}$

$= 9.78\times10^4$

$L = L_0/\gamma = 10^{-2}$ m

$\Rightarrow L_0 = 9.78\times10^4(10^{-2}\text{ m}) = 978$ m

(b) An observer on the bundle "sees" the accelerator shortened to 978 m from its proper length L_0

$L_0 = \gamma(978\text{ m}) = (9.78\times10^4)(978\text{ m}) = 9.57\times10^7$ m

(c) To apply the length contraction formula, we need to first go to the rest frame ~~where the~~ of the positron

$$L = \frac{\text{length of positron bundle in its rest frame}}{\gamma'}$$

where γ' = gama factor btw e^- & e^+ rest frames

From Problem 2-46

$$\gamma' = \frac{1}{\sqrt{1-\frac{u'^2}{c^2}}} = \frac{1+u^2/c^2}{1-u^2/c^2}$$

Since $\frac{u}{c} \simeq 1$, $\frac{u}{c} \simeq \sqrt{1-\frac{1}{\gamma^2}} \simeq 1-\frac{1}{2\gamma^2}$ eqn 2.40 of textbook

$\Rightarrow \left(\frac{u}{c}\right)^2 \simeq 1-\frac{1}{\gamma^2}$

$$\gamma' = \frac{1+u^2/c^2}{1-u^2/c^2}$$

$$= \frac{1+\left(1-\frac{1}{\gamma^2}+\cdots\right)}{1-\left(1-\frac{1}{\gamma^2}+\cdots\right)} \simeq 2\gamma^2$$

$$\Rightarrow \quad L = \frac{978\,m}{2\,(9.78\times10^4)^2} = 5\times10^{-8}\,m$$

(d) From before

$$E' = \gamma(E - v p_x) = \gamma(E + v|p_x|) = 9.78\times10^{6}\,GeV$$

$$p' = \gamma\left(p_x - \frac{v}{c^2}E\right) = -\gamma\left(|p_x| + \frac{v}{c^2}E\right) = -9.78\times10^{6}\,\frac{GeV}{c}$$

$\boxed{2\text{-}44}$ Center of mass frame of e^+e^- pair

$$E_{total} > 2M_ec^2 = 1.022\,MeV$$

$$p_{total} = 0$$

If e^+e^- pair is created from photon

$$p_\gamma = 0$$

$E_\gamma = p_\gamma c = 0 \Rightarrow$ energy is not conserved.

2-46 a) $u_x' = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}}$ $\qquad v = u \;,\; u_x = -u$

$$\Rightarrow u_x' = -\frac{2u}{1 + \frac{u^2}{c^2}}$$

$$\Rightarrow u' = \frac{2u}{1 + \frac{u^2}{c^2}} \qquad u' = \text{speed}$$

Next $$1 - \frac{u'^2}{c^2} = \frac{\left(1 + \frac{u^2}{c^2}\right)^2 - \left(\frac{2u}{c}\right)^2}{\left(1 + \frac{u^2}{c^2}\right)^2}$$

$$= \frac{\left(1 - \frac{u^2}{c^2}\right)^2}{\left(1 + \frac{u^2}{c^2}\right)^2}$$

$$\Rightarrow \boxed{\sqrt{1 - \frac{u'^2}{c^2}} = \frac{1 - u^2/c^2}{1 + u^2/c^2}}$$

b) Initial momentum in S' is due to moving particle

$$p'_{total} = \frac{m u'}{\sqrt{1 - \frac{u'^2}{c^2}}} = \frac{2mu}{1 + \frac{u^2}{c^2}} \cdot \frac{1 + u^2/c^2}{1 - u^2/c^2}$$

$$= \frac{2mu}{1 - \frac{u^2}{c^2}}$$

(c) Conservation of momentum

$$\Rightarrow \frac{Mu}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{2mu}{1-\frac{u^2}{c^2}}$$

$$\Rightarrow M = \frac{2m}{\sqrt{1-\frac{u^2}{c^2}}}$$

(d) In Reference frame S

$$E_i = \frac{2mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \qquad E_f = Mc^2$$

$$\Rightarrow E_f = Mc^2 = \frac{2mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \quad \text{from part (c)}$$

$$= E_i \quad ✓$$

In Reference frame S'

$$E_i' = mc^2 + \frac{mc^2}{\sqrt{1-\frac{u'^2}{c^2}}} \qquad E_f' = \frac{Mc^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Rightarrow E_i' = mc^2\left[1 + \frac{1+u^2/c^2}{1-u^2/c^2}\right] = \frac{2mc^2}{1-\frac{u^2}{c^2}} = \frac{Mc^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$= E_f' \quad ✓$$

2-49

(a) If ν mass is 0:

$$E_\mu^2 = (p_\mu c)^2 + (m_\mu c^2)^2 \qquad E_\nu^2 = (p_\nu c)^2$$

Conservation of energy:

$$\left(E_{K\mu} + m_\mu c^2\right) + E_\nu = m_\pi c^2$$

$$\Rightarrow E_{K\mu} + E_\nu = (139.56755 - 105.65839)\,\text{MeV}$$

$$\Rightarrow m_\mu^2 c^2(\gamma - 1) + E_\nu = 33.90916\ \text{MeV}$$

Conservation of momentum:

$$p_\mu = p_\nu$$

Combining conservation of energy with conservation of momentum

$$\Rightarrow \left(E_\mu^2 - m_\mu^2 c^4\right)^{1/2} = 33.90916\,\text{MeV} - m_\mu c^2(\gamma - 1)$$

$$\Rightarrow (\gamma^2 - 1)\, m_\mu^2 c^4 = (33.90916)^2 - 2(33.90916)\, m_\mu c^2 (\gamma - 1) + (m_\mu c^2)^2 (\gamma - 1)^2$$

$$\Rightarrow \gamma - 1 = \frac{(33.90916)^2}{2(m_\mu c^2)^2 + 2(33.90916)\, m_\mu c^2}$$

Substituting $m_\mu c^2 = 105.65839$ MeV

$$\Rightarrow \gamma - 1 = 0.0390 \quad \Rightarrow \gamma = 1.0390$$

$$E_{k\mu} = (\gamma - 1) m_\mu c^2 = 4.12 \text{ MeV} \quad \#$$

$$p_\mu = \frac{1}{c}\left[(\gamma m_\mu c^2)^2 - (m_\mu c^2)^2\right]^{1/2}$$

$$= \sqrt{\gamma^2 - 1} \cdot \frac{105.65839 \text{ MeV}}{c} = 29.8 \frac{\text{MeV}}{c}$$

$\#$

$$E_\nu = c p_\nu = 29.8 \text{ MeV} \quad \#$$

(b.) $m_\nu = 190\,\text{keV}$

$m_\pi c^2 = E_\mu + E_\nu$ conservation of energy

$p_\nu = p_\mu$ conservation of momentum

$$\Rightarrow \frac{E_\nu^2}{c^2} - m_\nu^2 c^2 = \frac{E_\mu^2}{c^2} - m_\mu^2 c^2$$

$$\Rightarrow (E_\nu - E_\mu)(E_\nu + E_\mu) = (m_\nu^2 - m_\mu^2)c^4$$

$$E_\nu - E_\mu = \frac{(m_\nu^2 - m_\mu^2)c^2}{m_\pi}$$

Together with conservation of energy

$$2E_\nu = \frac{m_\nu^2 - m_\mu^2}{m_\pi} c^2 + m_\pi c^2$$

$$\Rightarrow E_\nu = \frac{1}{2}\left(\frac{m_\nu^2 - m_\mu^2}{m_\pi} c^2 + m_\pi c^2\right) = 29.8\,\text{MeV}$$

$$E_\mu = \frac{1}{2}\left(m_\pi c^2 + \frac{m_\mu^2 - m_\nu^2}{m_\pi} c^2\right) = 109.8\,\text{MeV}$$

$$p_\nu = \sqrt{\frac{E_\nu^2}{c^2} - m_\nu^2 c^2} = 29.8\,\text{MeV}$$

$$p_\mu = \sqrt{\frac{E_\mu^2}{c^2} - m_\mu^2 c^2} = 29.7\,\text{MeV}$$