Problem set 3

Assigned: September 22, 2006

due: Friday, September 29 at 5 PM in the box near 2103 Chamberlin

Problems

1. TM 4-83

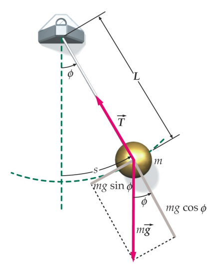
2. TM 5-42

3. TM 5-43

4. TM 5-80

5. TM 5-95

6. Consider the motion of the pendulum shown below where all the forces on the mass at the end has been drawn.



a) What is the length s as a function of ϕ?

b) Using the forces indicated on the diagram, write Newton's equation motion governing ϕ. (i.e. The "F=ma" equation.)

c) Assuming that ϕ is small (i.e. $\phi \ll 1$), Taylor expand $\sin\phi$ appearing in part b) about $\phi = 0$ to linear order in ϕ (i.e. first order in ϕ).

d) Assuming that the mass is initially released from rest with the angle ϕ initially having a value $\phi_i \ll 1$, find ϕ as a function of time using the result of part c). (Hint: recall the mass at the end of a spring example from lecture.)

7. Suppose a small planet with mass m_2 is in a circular orbit with radius r around a very massive star with mass m_1 which is assumed to not move due to its large mass. Treat both the star and the planet as pointlike masses, and use polar coordinates to describe the plane of the orbit with the star sitting at $r = 0$.

a) Using Newton's second law and law of gravity, show

$$\frac{d^2\hat{r}}{dt^2} = -\frac{Gm_1}{r^3}\hat{r} \tag{1}$$

where $\hat{r}$ is the radial vector direction.

b) To solve for $\hat{r}(t)$, one can write without loss of generality

$$\hat{r}(t) = x_u(t)\hat{x} + y_u(t)\hat{y}$$

and take a dot product of both sides of the equation (1) with respect to $\hat{x}$ and $\hat{y}$ to obtain an equation for $x(t)$ and $y(t)$. i.e.

$$\hat{x} \cdot \frac{d^2\hat{r}}{dt^2} = -\frac{Gm_1}{r^3}\hat{r} \cdot \hat{x} \longrightarrow \frac{d^2x_u}{dt^2} = \frac{-Gm_1}{r^3}x_u.$$

$$\hat{y} \cdot \frac{d^2 \hat{r}}{dt^2} = -\frac{Gm_1}{r^3} \hat{r} \cdot \hat{y} \longrightarrow \frac{d^2 y_u}{dt^2} = \frac{-Gm_1}{r^3} y_u.$$

Solve for $x_u(t)$ if you are given that $x_u(t=0) = r$ and $\dot{x}_u(t=0) = 0$. (Hint: recall the mass at the end of a spring example from lecture).

c) What is the period of this orbit?