HW #4

1-6 Yes, you will see your image and it will look as it does now. The reason is the second postulate: All observers have the same light speed. Light reflects from you to the mirror at speed c relative to you and ~~the~~ reflects from the mirror back to you at speed c, independent of your motion

1-8 a) No

c) Yes, due to Einstein's 2nd postulate

d) No

1-26 a) $t = \frac{4\ c \cdot years}{0.75c} = \frac{4\ years}{0.75} = 5.33\ years$ ✲

b) For passenger, length contraction

$L = 4\ c\ years \sqrt{1-\beta^2} = 4\ c\ years \sqrt{1-0.75^2}$

$= 2.65\ c \cdot years$

$\Rightarrow t = \frac{2.65\ c\ years}{0.75\ c} = 3.53\ years$ ✲

1-28 In S' frame: $\Delta x' = 1\ m \cos 30^\circ = 0.866\ m$

$\Delta y' = 1\ m \sin 30^\circ = 0.5\ m$

Length contraction $\Delta x = \Delta x' \sqrt{1-\beta^2} = 0.866\ m \sqrt{1-0.8^2}$

$= 0.52\ m$

$\Delta y = \Delta y' = 0.5\ m$

Hence $\tan\theta = \frac{\Delta y}{\Delta x} = \frac{0.5}{0.52}$

$\Rightarrow \boxed{\theta = 43.9^\circ}$

$L = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{0.5^2 + 0.52^2}\ m = \boxed{0.721\ m}$

1-41 Orbit circumference $= 4 \times 10^7$ m

Satellite speed $= \dfrac{4 \times 10^7 \text{ m}}{90 \text{ min} \times 60 \text{ s/min}} = 7.41 \times 10^3 \text{ m/s}$

$\Delta t - \Delta t_0 = t_{diff}$

$\Delta t \left(1 - \frac{1}{\gamma}\right) = t_{diff}$

$\Rightarrow \Delta t \left(\frac{\beta^2}{2}\right) \simeq t_{diff}$

$\Rightarrow t_{diff} = (1 \text{ year}) \times \frac{1}{2} \left(\frac{7.41 \times 10^3 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right)^2$

$\simeq (\pi \times 10^7 \text{ s}) \frac{1}{2} \left(\frac{7.41 \times 10^3 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right)^2$

$\simeq 9.6 \text{ ms} = \boxed{9.6 \times 10^{-3} \text{ s}}$

1-43 a) $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \boxed{2.55}$

b) $\tau = 2.6 \times 10^{-8} \text{ s} \Rightarrow \tau_{lab} = \gamma \tau = \boxed{6.63 \times 10^{-8} \text{ s}}$

(c) $N(t) = N_0 e^{-t/\tau}$ In pion's frame,

$L = L_0 \sqrt{1-\beta^2} = 19.6 \text{m}$

$t = \dfrac{L}{v} = \dfrac{19.6 \text{m}}{0.92 c} = 7.1 \times 10^{-8} \text{ s}$

So, if $N_0 = 50{,}000$

$$\Rightarrow N(t) = 5\times10^4\, e^{-7.1/2.6} = \boxed{3260}$$

(d) Ignoring relativity

$$t = \frac{L}{v} = \frac{50\,m}{0.92c} = 1.81\times10^{-7}\,s$$

$$\Rightarrow N(t) = 5\times10^4\, e^{-18.1/2.6} = \boxed{47}$$

1-46 a) $L = L_0\sqrt{1-\beta^2} = 100\,m\sqrt{1-0.85^2} = \boxed{52.7\,m}$

(b) $u' = \dfrac{u+u}{1+u\cdot u/c^2} = \dfrac{0.85c+0.85c}{1+0.85^2} = \boxed{0.987c}$

(c) $L = L_0\sqrt{1-0.987^2} = \boxed{16.1\,m}$

(d) $\Delta t = \dfrac{L}{u} = \dfrac{52.7\,m}{0.85\,c} = \boxed{2.1\times10^{-7}\,s}$

(e)