P247 HW # 6 Solutions

6-33

$$\text{Work} = wp \quad \text{in 1 turn}$$

$$= F \cdot 2\pi R$$

$$\Rightarrow \quad F = \frac{wp}{2\pi R}$$

Mechanical advantage

$$= \frac{w}{F} = \frac{w}{\frac{wp}{2\pi R}}$$

$$= \boxed{\frac{2\pi R}{p}}$$

$\boxed{6\text{-}45}$

(a) $\frac{d}{dt}(\vec{r}\cdot\vec{r}) = 2\vec{v}\cdot\vec{r} = 0 \Rightarrow \vec{v}\cdot\vec{r} = 0$, ie. $\vec{v}\perp\vec{r}$

(b) $\frac{d}{dt}(\vec{v}\cdot\vec{v}) = 2\vec{a}\cdot\vec{v} = 0 \Rightarrow \vec{a}\cdot\vec{v} = 0$, ie. $\vec{a}\perp\vec{K}$

If $\vec{a}\perp\vec{v}$ & $\vec{r}\perp\vec{v}$ then $\vec{a}\parallel\vec{r}$ (in 2D)

(c) $\frac{d}{dt}(\vec{v}\cdot\vec{r}) = \vec{a}\cdot\vec{r} + V^2 = 0 \Rightarrow \boxed{a_r = -\frac{v^2}{r}}$

$\boxed{6\text{-}53}$ $V = (V_0 - gt)\hat{y}$ constant acceleration

$$\int \vec{F} \cdot \vec{v}\, dt = \int_0^t (-mg)(V_0 - gt)\, dt$$

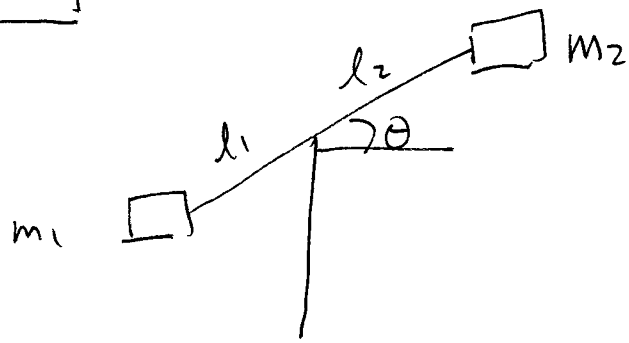
$$= -mgV_0 t + \frac{mg^2t^2}{2}$$

$$K = \frac{1}{2} mv^2 = \frac{1}{2} m (V_0 - gt)^2$$

$$K - K_0 = \frac{1}{2} m (V_0 - gt)^2 - \frac{1}{2} m V_0^2$$

$$= -mgV_0 t + \frac{mg^2t^2}{2}$$ identical to $\int \vec{F} \cdot \vec{v}\, dt$!

$\boxed{6-61}$



(a) $U = m_2 g l_2 \sin\theta - m_1 g l_1 \sin\theta = (m_2 l_2 - m_1 l_1) g \sin\theta$

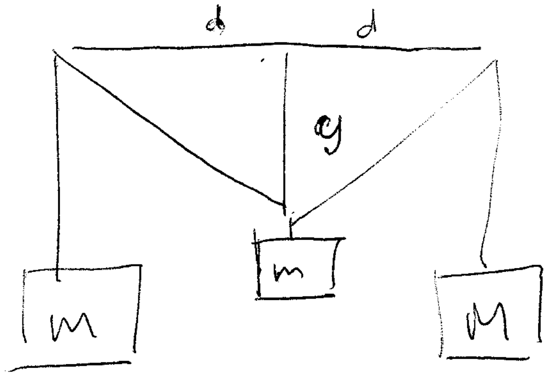
(b) Minimum at $\sin\theta = \begin{cases} -1 & \text{if } m_2 l_2 > m_1 l_1 \\ 1 & \text{if } m_1 l_1 > m_2 l_2 \end{cases}$

i.e. orients vertically with heavier mass at bottom, as expected

(c) If $m_2 l_2 = m_1 l_1$, then $U = 0$ Independent of θ

neutral equilibrium

6-71



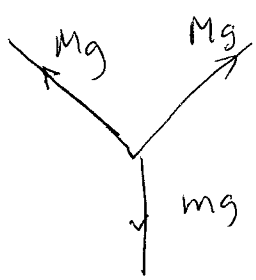
(a) $U = mgy - 2Mg\sqrt{d^2+y^2}$

(b) $\dfrac{du}{dy} = mg - 2Mg\,\dfrac{\frac{1}{2}\cdot 2y}{\sqrt{d^2+y^2}} = 0 \qquad (*)$

$\Rightarrow \quad m^2(d^2+y^2) = 4M^2y^2$

$\Rightarrow \quad \boxed{y = \dfrac{md}{\sqrt{4M^2+m^2}}}$

(c)



$0 = mg - \dfrac{2Mgy}{\sqrt{d^2+y^2}}$ same as $(*)$

(d) $\dfrac{d^2u}{dy^2} = \dfrac{d}{dy}\left(mg - \dfrac{2Mgy}{\sqrt{d^2+y^2}}\right) = \dfrac{-2Mg}{\sqrt{d^2+y^2}} + \dfrac{2Mgy^2}{(d^2+y^2)^{3/2}}$

$= -2Mg\dfrac{d^2}{(u^2+d^2)^{3/2}} < 0 \quad \therefore$ stable

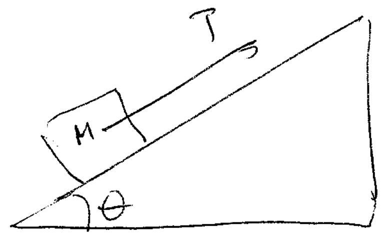
$\boxed{6-72}$

$$mgh = \frac{1}{4} \cdot 60.7 \times 10^9 \ kW \cdot h \times 10^3 \frac{W}{kW} \times 3600 \frac{sec}{h}$$

$$h = \frac{2.2 \times 10^{17} J}{4 (60kg)(9.8 m/s^2) 287 \times 10^6}$$

$$= 3.2 \times 10^5 m$$

$\boxed{6-86}$



$$ma = T - mg\sin\theta$$

(a) $W = Tx$

(b) $ma = T - mg\sin\theta$

Work energy theorem $(T - mg\sin\theta)X = \Delta K = \frac{1}{2}mv^2$

$$\Rightarrow V = \sqrt{\left(\frac{2T}{m} - 2g\sin\theta\right)X}$$

(c) $P = TV = T\sqrt{\left(\frac{2T}{m} - 2g\sin\theta\right)X}$

$\boxed{6\text{-}89}$

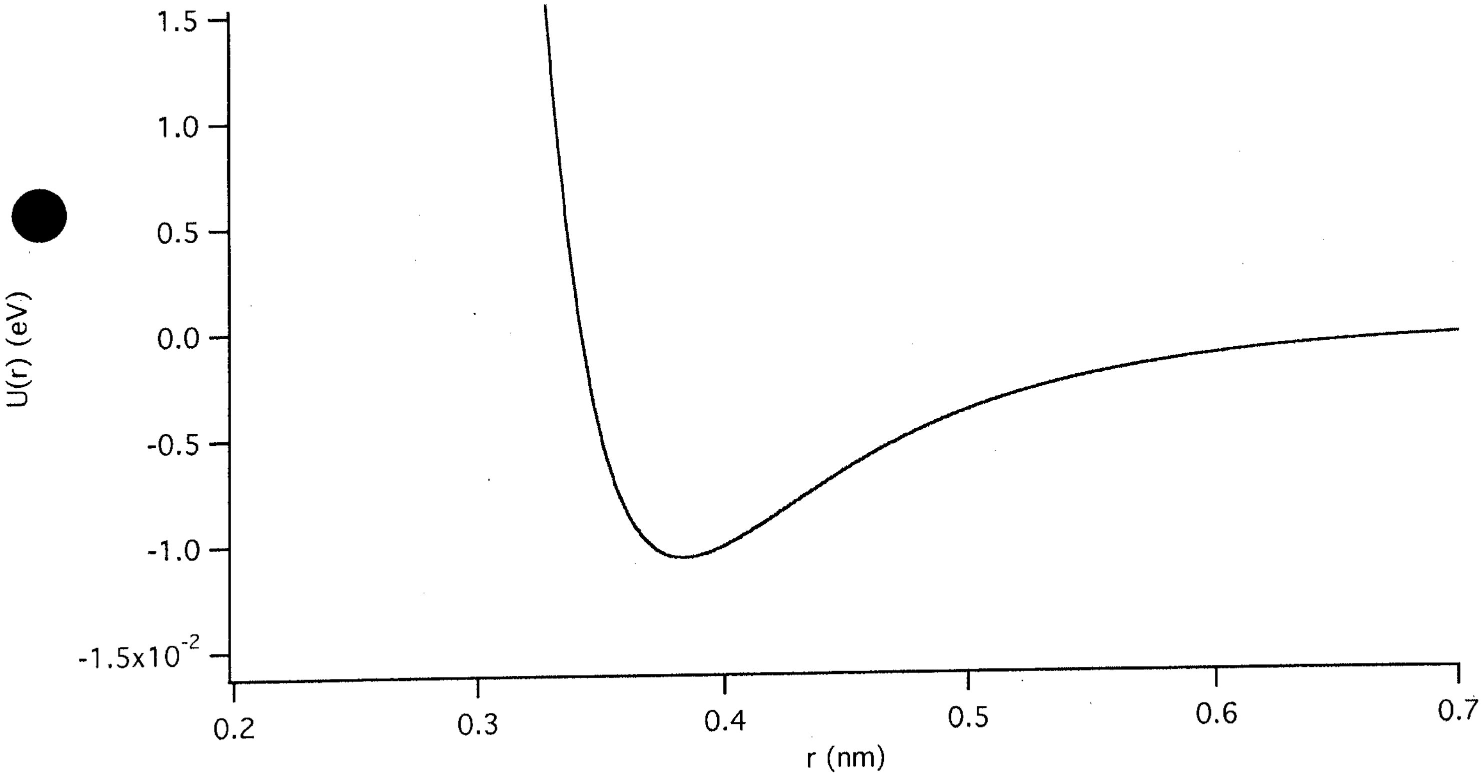
(a) See attached graph. U is indeed repulsive for small r, attractive for large r

(b) Stable equilibrium at $r = 0.384\,nm$ $U = -0.0107\,eV$

(c) $r = 5\,\text{Å}$ $F = -0.042\frac{eV}{nm} \times 1.6\times10^{-19}\frac{J}{eV} \cdot \frac{10^{9}\,nm}{m}$

$= -6.7 \times 10^{-12}\,N$

$r = 3.5\,\text{Å}$ $F = 0.468\,\frac{eV}{nm} = 7.46\times10^{-11}\,N$


1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5x10^{-2}
U(r) (eV)
0.2
0.3
0.4
0.5
0.6
0.7
r (nm)