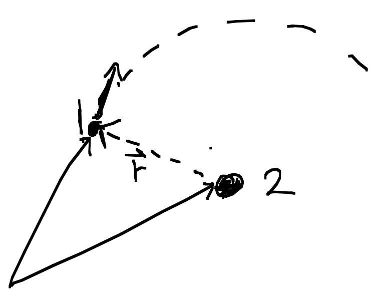
Note Title 9/26/2006

Lecture 10

Suppose the forc e law of a new kind of particle is



$$\vec{F}_{12} = -\Lambda \frac{m_1^2 m_2}{r^3} \hat{r}$$

force on 1 acted on by 2

Note these are different from Newton's Law.

Find the period of the orbit assuming 2 is not moving

$$\cancel{m_1} \frac{d^2\vec{r}}{dt^2} = -\Lambda \frac{m_1^{\cancel{2}} m_2}{r^3} \hat{r}$$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} \qquad \frac{d^2\vec{r}}{dt^2} = \frac{d^2r}{dt^2}\hat{r} + 2\frac{dr}{dt}\frac{d\hat{r}}{dt} + r\frac{d^2\hat{r}}{dt^2}$$

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta} \qquad \frac{d^2\hat{r}}{dt^2} = \ddot{\theta}\hat{\theta} + \dot{\theta}\frac{d\hat{\theta}}{dt}$$

$$= \ddot{\theta}\hat{\theta} - \dot{\theta}^2\hat{r}$$

$$\therefore \quad \frac{d^2r}{dt^2}\hat{r} + 2\frac{dr}{dt}\dot{\theta}\hat{\theta} + r(\ddot{\theta}\hat{\theta} - \dot{\theta}^2\hat{r}) = -\Lambda\frac{m_1m_2}{r^3}\hat{r}$$

$$\frac{d^2r}{dt^2} - r\dot{\theta}^2 = -\Lambda\frac{m_1m_2}{r^3}$$

$$2\frac{dr}{dt}\dot{\theta} + r\ddot{\theta} = 0$$

Since circular orbit $\therefore$ $\frac{dr}{dt} = 0 \qquad \frac{d^2r}{dt^2} = 0$

$$\Rightarrow \ddot{\theta} = 0 \Rightarrow \boxed{\dot{\theta} = \text{const}} \equiv \omega$$

$$-r\omega^2 = -\Lambda \frac{m_1 m_2}{r^3}$$

$$\boxed{\omega = \sqrt{\Lambda \frac{m_1 m_2}{r^4}}} \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$$

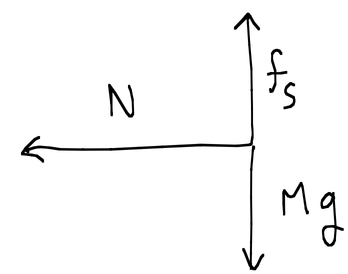
example Amusement park ride

If the drum spins fast enough, the person will be stuck to the wall. What is that spin rate?

person w/ mass M

Force diagram:

Since not falling,



Let's compute f_s: Need normal force

$$
\begin{aligned}
N &= \text{centripetal force.} \\
&= M a_c \\
&= M \frac{v^2}{R} = M \frac{(R\omega)^2}{R} \\
&= M R \omega^2
\end{aligned}
$$

$$f_s \le \mu_s N \le \mu_s M R \omega^2$$

Since $f_s - Mg = 0$,

$$\cancel{M} g \le \mu_s \cancel{M} R \omega^2$$

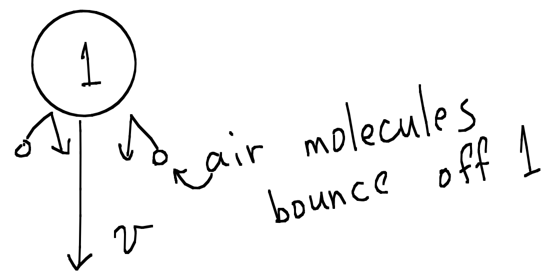
$$\boxed{\sqrt{\frac{g}{\mu_s R}} \le \omega}$$

Cloth on wood $\mu_s \gtrsim 0.3$

If the drum has a radius of 6 ft, $\boxed{\omega \gtrsim 4 \ \frac{\text{rad}}{s}}$

$$\Rightarrow \boxed{f = \frac{\omega}{2\pi} = 6 \text{ turns/s}}$$

Drag force:

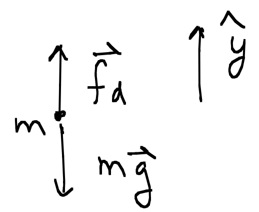


$\Longrightarrow$ force against the direction of the fall.

$$\boxed{\vec{f}_d = -b|v|^n \hat{v}}$$ drag force

example

find the motion of the ball.



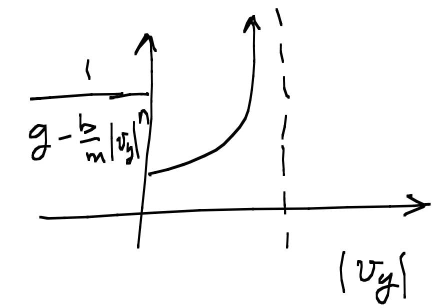
$\vec{v} = v_y \hat{y}$ $\quad -\hat{v} = +\hat{y}$ since going down.

$$m \frac{dv_y}{dt} = b|v_y|^n - mg \Longrightarrow \boxed{\int_{v_i}^{v_f} \frac{dv_y}{\frac{b}{m}|v_y|^n - g} = \int_{t_i}^{t_f} dt}$$

Since falling $v_{y_f} < v_{y_i}$ (e.g $-60\frac{m}{s} < 0\frac{m}{s}$)

$\therefore$ better to rewrite the integral as

$$\int_{v_{y_f}}^{v_{y_i}} \frac{dv_y}{g - \frac{b}{m}|v_y|^n} = t_f - t_i$$



If one waits a long time, $t_f - t_i \to \infty$.

$\therefore \quad b|v_y|^n \longrightarrow mg$

Hence, if for example initially at rest, $v_{y_i} = 0$,
$|v_y|$ increases approaching $\boxed{|v_y| \longrightarrow \left(\frac{mg}{b}\right)^{1/n}}$ terminal speed Can't go faster