Lecture 11

Note Title 9/29/2006

Exam questions

1) Newton's second law + $\vec{a} = \frac{d^2\vec{x}}{dt^2}$ $\vec{v} = \frac{d\vec{x}}{dt}$ + integration

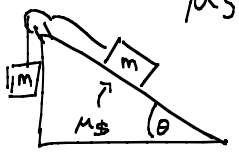
2) 3 multiple choice questions
   i) pulley + "inertia" concept
   ii) uniform acceleration in 1D
   iii) derivatives of polar coordinate basis vectors

3) $\vec{F} = m\vec{a}$ and circular motion

4) Inclined plane + friction + tension
+ 1D kinematics

Example

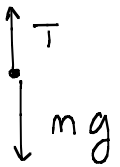
$$\mu_s = \frac{1}{\sqrt{3}}$$

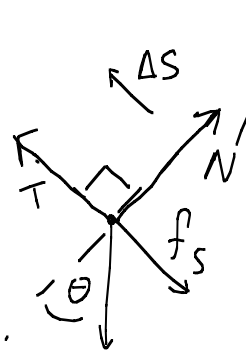


Everything initially at rest. θ is slowly decreased.

At a critical angle $\theta = \theta_c$ the masses start to slide. Find that angle.

ANS





$f_s = (mg\cos\theta)\mu_s$ since max just before moving

$ma = -mg\sin\theta + T - f_s$

$ma = mg - T$

$$\Rightarrow \quad ma = -mg\sin\theta + (mg - ma) - (mg\cos\theta)\mu_s$$

$$a = -g\sin\theta + g - a - g\cos\theta\,\mu_s$$

$$\therefore \quad a = \frac{1}{2} g \left(1 - \mu_s \cos\theta - \sin\theta\right)$$

$$= 0$$

$$\therefore \quad 1 - \mu_s \cos\theta - \sin\theta = 0$$

$$\boxed{1 = \mu_s \cos\theta + \sin\theta}$$

$$\mu_s = \frac{1}{\sqrt{3}}$$

at $\theta_c = \frac{\pi}{6}$, $\quad \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{2} + \frac{1}{2} = 1$

$$\Rightarrow \boxed{\theta_c = \frac{\pi}{6}}$$