PHY 247 Lecture 16 Gary Shiu 10/13/06

Recap: ① Time dilation $\Delta t = \gamma \Delta t'$

② Length contraction $L' = \frac{L}{\gamma}$

③ Muon decay as an example

However, time dilation / length contraction formulae above are applicable only if one of the ref. frames ~~one~~ is special (proper time, rest frame, ...)

In general, transformation btn ref. frames requires:

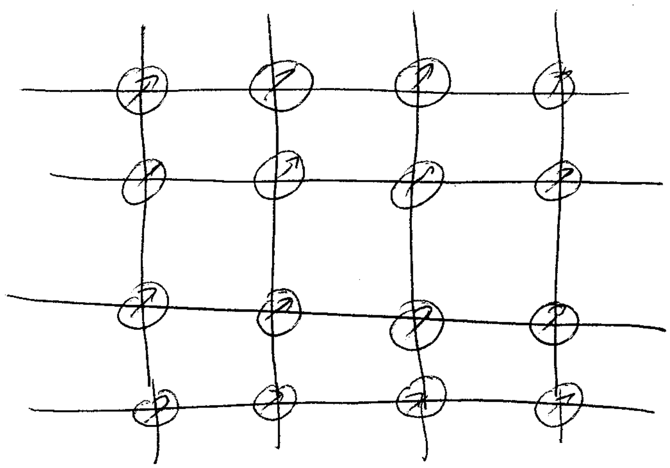
Lorentz transformation

Today ① Synchronizing clocks in relativity

② Lorentz transformation

Synchronizing Clocks

Imagine an inertial frame being a grid of points with a clock located at each point



This array of clocks allow us to describe spacetime events

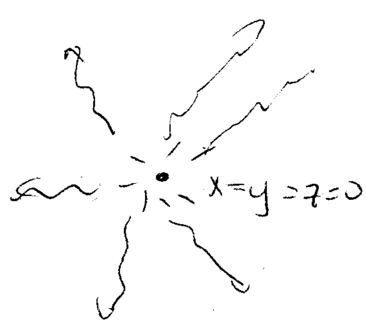
Q: Can we look at a clock somewhere else to record the time of an event? why?

Next question: How do we synchronize the clocks?

Remember: light has a speed limit

Take the clock at origin $x = y = z = 0$

Set clock at origin to be $t = 0$ at the instant we send out a light flash.



When someone at $\vec{r}$ receives light signal, set $t = \frac{|\vec{r}|}{c} = \frac{\sqrt{x^2+y^2+z^2}}{c}$

read same time as clock at origin

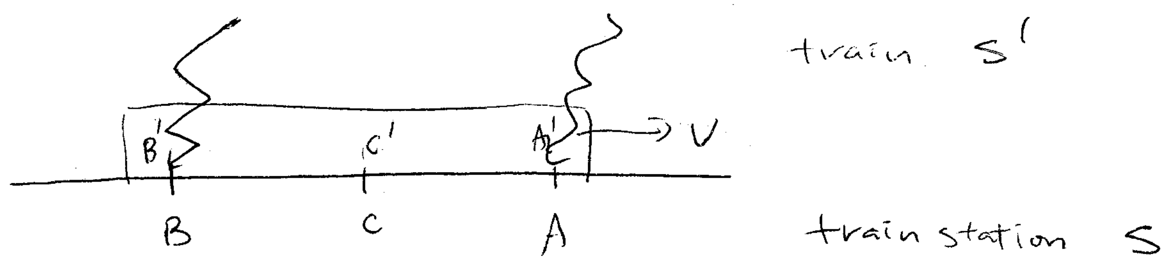
Relativity of Simultaneity

Now that we have synchronized the clocks, we can ask whether 2 events are simultaneous.

Two events simultaneous in S are not necessarily simultaneous in S'.

[More precisely, if two events are spatially separated in S

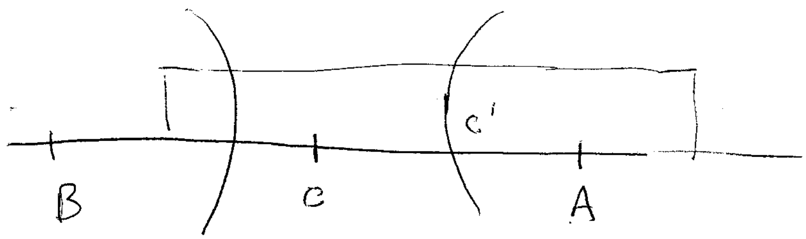
Einstein's Example



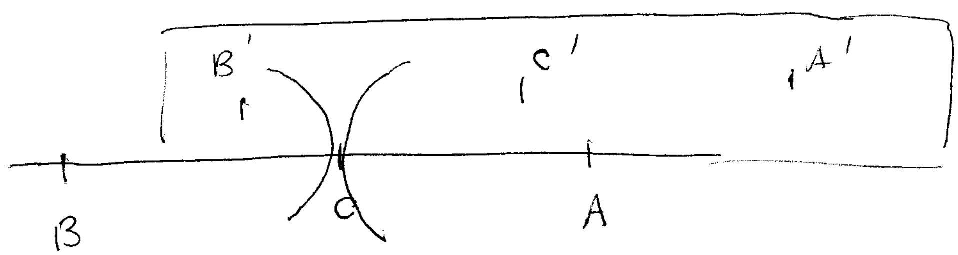
Lightning hits A & B simultaneously in S, i.e. flash from A & B reaches midpt C at the same time in S-frame

Not so for ref. frame S' of train

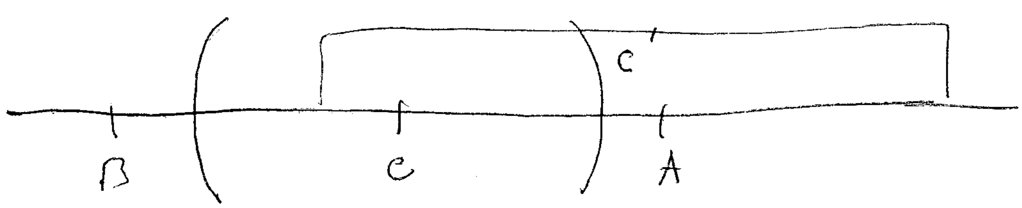
[C' does not record the flash from A' & B' at the same time in S' frame]



Flash from front end of train reaches C' first



Even when light reaches C, flash from rear end of train has not reached C'



Eventually, flash from rear end of train reaches C'

To put Einstein's postulates into practical use, we will introduce 2 useful tools:

1. Lorentz transformation (Lecture)
2. Spacetime Diagram (Lab)

Lorentz transformation

Einstein's 2nd postulate is inconsistent with Galilean transformation.

$$u_x' = c - v \quad \text{rather than } c$$

Need to modify Galilean transformation

Since Galilean transformation works well at low velocity, such modification must reduce to Galilean transformation when $v \ll c$

Assume:

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

where γ depends on v not position [no preferred origin]

Require $\gamma \to 1$ as $\frac{v}{c} \to 0$

The inverse transformation must look exactly the same, except for the sign of v

$$x = \gamma\,(x' + vt')$$

From transf. of x, can deduce transf of t

$$t' = \gamma \left[t + \frac{1-\gamma^2}{\gamma^2} \frac{x}{v} \right]$$

Send out a flash of light from origin of S at $t=0$

$$x^2 + y^2 + z^2 = c^2 t^2$$

spherical wave with speed $= c$

The origin of S & S' coincide at $t=0$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

[same c due to relativity]

$$\Rightarrow \gamma^2 (x - vt)^2 + y^2 + z^2 = c^2 \gamma^2 \left[t + \frac{1-\gamma^2}{\gamma^2} \frac{x}{v} \right]^2$$

Consistent if this is identical to $x^2 + y^2 + z^2 = c^2 t^2$

$\therefore$ coefficient of $x^2 = 1$

" " $t^2 = c^2$

" " $xt = 0$

Hence : $$\gamma^2 - c^2\gamma^2 \frac{(1-\gamma^2)^2}{\gamma^4 v^2} = 1$$

$$\Rightarrow \quad -c^2 \frac{(1-\gamma^2)^2}{\gamma^2 v^2} = 1-\gamma^2$$

$$\Rightarrow \quad \boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Lorentz transformation

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y$$
$$z' = z$$

Check

Consider two events (x_1, t_1), (x_2, t_2) in frame S

$$\Delta t = t_2 - t_1$$

$$\Delta t' = t_2' - t_1' = \gamma(t_2 - t_1) - \frac{\gamma v}{c^2}(x_2 - x_1)$$

$$= \gamma \Delta t - \frac{\gamma v}{c^2}\Delta x$$

depends on whether the events happen at the same time

- Clock synchronized in frame S, not necessarily in S'

- proper time, $\Delta x' = 0$

$$\Rightarrow \quad \Delta t = \gamma \Delta t'$$

- Can two events which happen at different times appear to be simultaneous in frame S'?

$$0 = \Delta t' = \gamma \Delta t - \frac{\gamma v}{c^2} \Delta x$$

$$\Rightarrow \quad \beta \equiv \frac{v}{c} = \frac{c \Delta t}{\Delta x} \qquad \text{if} \quad \frac{c \Delta t}{\Delta x} \leq 1$$