PHYSICS 247 Lecture 17 — Gary Shiu — 10/16/06

Recap: Lorentz Transformation

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$y' = y$$
$$z' = z$$

Today:
- Velocity Transformation
- Spacetime diagram (very brief)
- Spacetime interval
- Doppler's effect (if time permits)

Velocity Transformation

$$dx' = \gamma(dx - v\,dt)$$

$$dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right)$$

$$dy' = dy$$

$$dz' = dz$$

Hence $u_x' = \dfrac{dx'}{dt'} = \dfrac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)} = \dfrac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$

$$= \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)} = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

$$u_z' = \frac{dz'}{dt'} = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

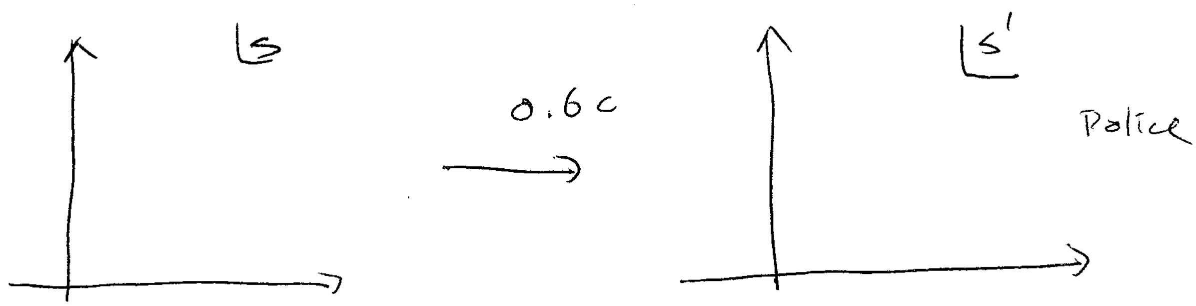
Note

1. $\frac{v}{c} \ll 1 \rightarrow$ Galilean transformation
2. Not only u_x' but u_y', u_z' also changes from frame to frame
3. Einstein's postulate: $u_x = c \Rightarrow u_x' = c$

<u>Example</u> OJ is driving a white SUV at 0.9c

A Police car is chasing him at 0.6c, on which a policeman shoots a bullet towards OJ at 0.6c.

Can OJ get away?



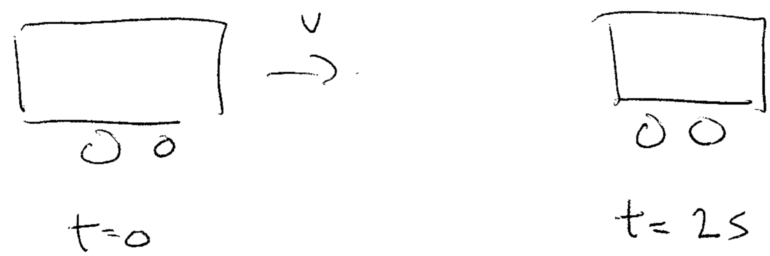
$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{0.6c + 0.6c}{1 + (0.6)^2} = 0.88c$$

$$< 0.9c$$

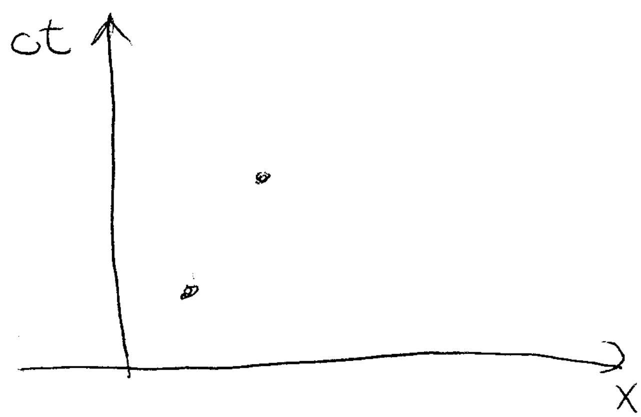
$\Rightarrow$ OJ can get away ...

Spacetime Diagram

Instead of drawing snapshots of events:



would be nice to put all the events in 1 picture



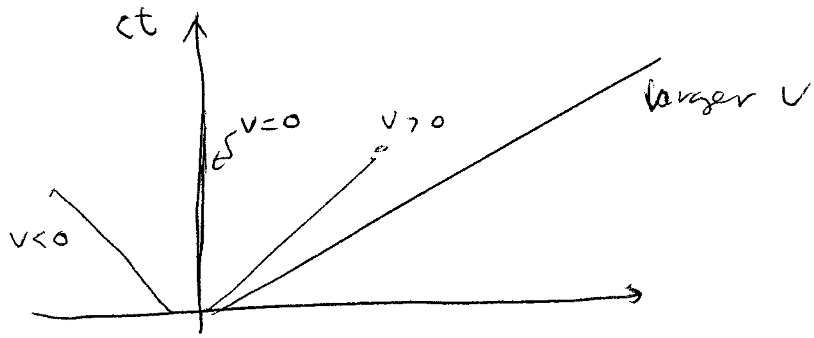
choose ct so that points won't clutter if we consider high vel

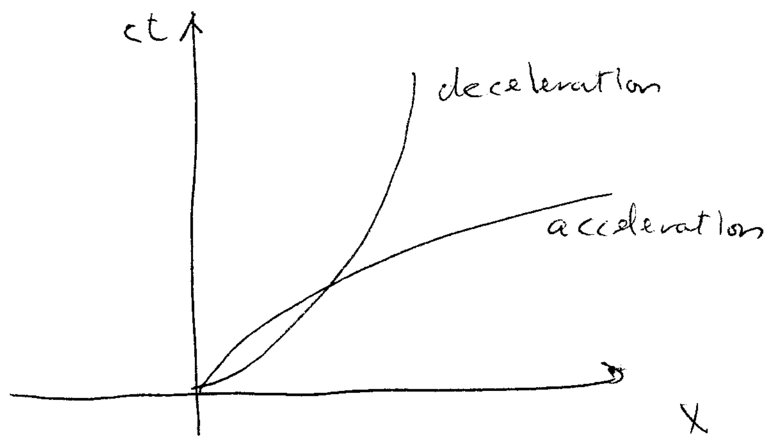
Limited by our ability to visualize higher dim ⇒ 2D

[Reason we have cleverly chosen x', t' to be the only coordinates that change. We can suppress y', z']

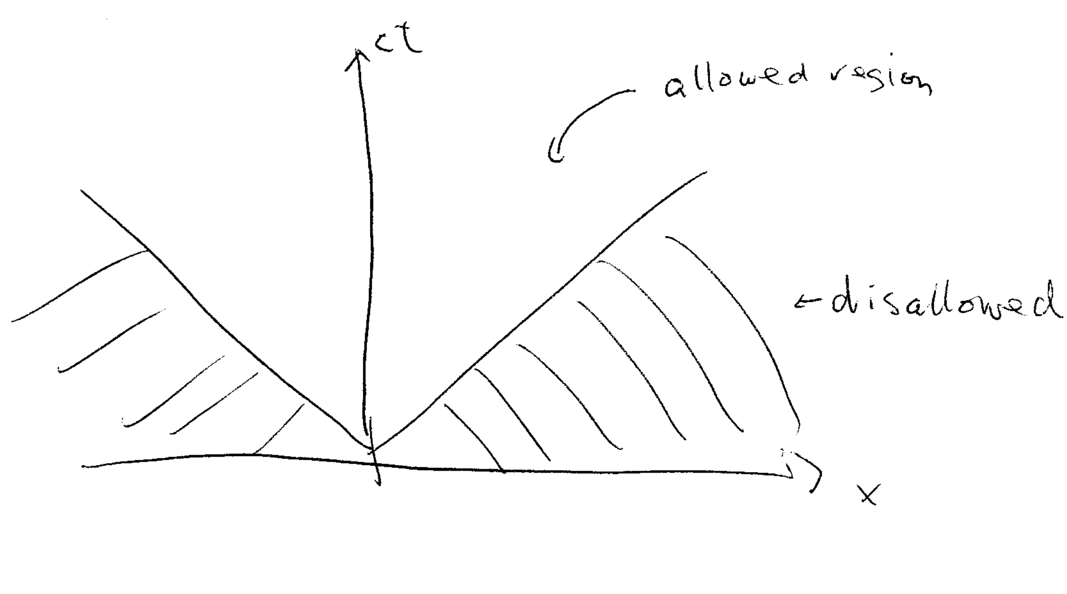
Worldline : trajectory on spacetime diagram

Three key points

1. 

2. 

3. Speed limit = c



Spacetime Interval

In relativity : length, time, simultaneity depends on frame of reference

Are there quantities that are invariant ?

Yes : spacetime interval

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

$$= (\text{time separation})^2 - (\text{space separation})^2$$

Important to distinguish 3 cases:

$$(\Delta s)^2 = \begin{cases} > 0 & \text{timelike} \quad \text{particle trajectory} \\ = 0 & \text{lightlike} \quad \text{light travels} \\ < 0 & \text{spacelike} \quad \text{casually disconnected} \end{cases}$$

- Two events separated by time-like interval can communicate (say by light signal)

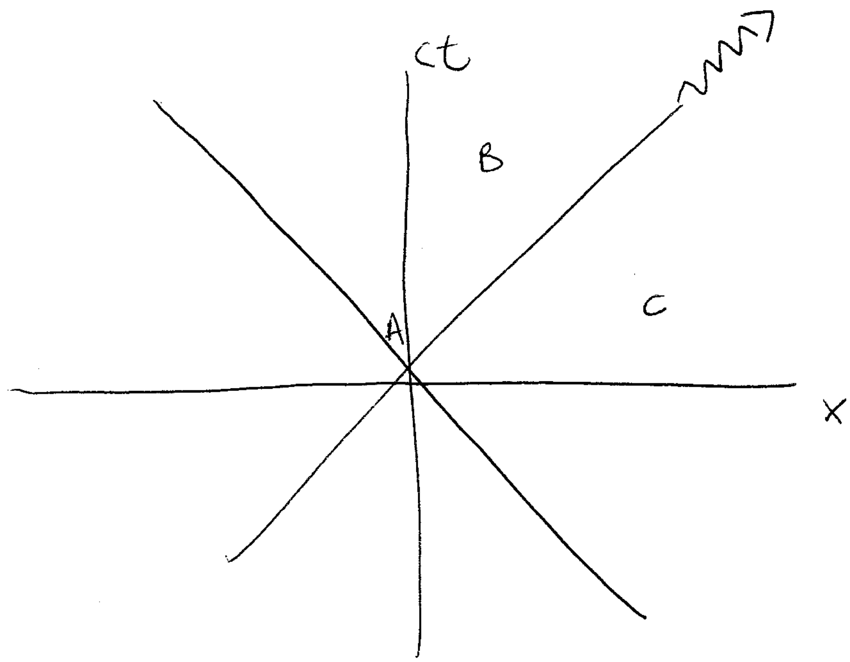
  e.g. the birth of you and your grandpa

  Their orderings are the same in all ref. frames

- Two spacelike events cannot affect one another casaully. (no signal can travel fast enough)

  Their orderings depend on the ref frame

Spacetime Diagram



A & B are casaully connected

A & C are casaully disconnected

Exercise: Find another reference frame such that C happens **before** A.