PHY 247 Lecture 18 Gary Shiu 10/18/06

Recap: - Velocity transformation
- spacetime diagram (useful for conceptual questions, not in details such as calibration)
- Spacetime Interval (casaulity)

Today - Doppler's effect (applications: Hubble's law)
- Twin Paradox (more paradoxes in lab)

Doppler's Effect

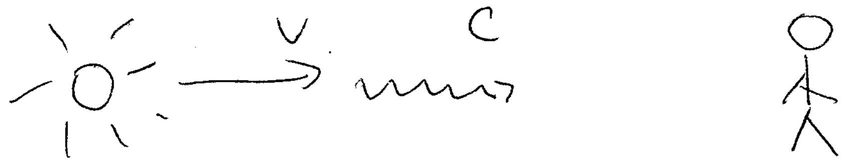
Also exists for sound (siren of an ambulance)

<u>but</u>

change in frequency depends on whether source or receiver is moving because sound propagates in a medium (air)

Different for light : Einstein's postulate

$\Rightarrow$ Need to derive Doppler's formula from scratch!



Source emits N waves in Δt

Wave travels distance $= c\Delta t$ (why?)

Source " " $= v\Delta t$

According to observer  , train of N waves occupies a distance $c\Delta t - v\Delta t$

Wavelength $= \lambda = \dfrac{c\Delta t - v\Delta t}{N}$

Frequency $= f = \dfrac{c}{\lambda} = \dfrac{cN}{(c-v)\Delta t}$

Relate f to the proper frequency
↓ rest frame of source

$$f_0 = \frac{N}{\Delta t'} = \frac{\gamma N}{\Delta t} \qquad \text{time dilation}$$

$$\Rightarrow \qquad f = \frac{1}{1-\beta}\,\frac{N}{\Delta t} \qquad \beta \equiv \frac{v}{c}$$

$$= \frac{f_0}{(1-\beta)\gamma}$$

← relativity effect $\sim \beta^2$
(in contrast β effect in usual Doppler's effect)

Relativistic Doppler effect differs from usual by 2nd order effects in $\frac{v}{c}$

$$\boxed{f = \frac{\sqrt{1-\beta^2}}{1-\beta}\, f_0 = \sqrt{\frac{1+\beta}{1-\beta}}\, f_0}$$

Source approaching observer

$f > f_0$ "blueshift"

Similarly

Replace $c\Delta t - v\Delta t$ by $c\Delta t + v\Delta t$ in derivation

$$\boxed{f = \frac{\sqrt{1-\beta^2}}{1+\beta} f_0 = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0} \qquad \text{receding}$$

$$f < f_0 \qquad \text{"redshift"}$$

For small $\frac{v}{c} \equiv \beta$, use Taylor expansion

$$f = \begin{cases} f_0 \left(1+\frac{1}{2}\beta+\cdots\right)\left(1+\frac{1}{2}\beta+\cdots\right) & \text{blueshift} \\ f_0 \left(1-\frac{1}{2}\beta+\cdots\right)\left(1-\frac{1}{2}\beta+\cdots\right) & \text{redshift} \end{cases}$$

$$= f_0(1 \pm \beta) \qquad \begin{matrix} + & \text{approaching} \\ - & \text{receding} \end{matrix}$$

However, astronomical source can move very fast away from us, use <u>exact</u> expression

Hubble's law: distant object appears to be receding from us with velocity ~ distance

[deduce from measuring spectral lines]

Hubble's law $\quad v = \underset{\substack{\uparrow \\ \text{Hubble constant}}}{H}\, r$

Redshift factor

$$\boxed{z = \frac{f_0 - f}{f}}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

$$\boxed{\beta = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}}$$

Not unusual to talk about $z \sim$ a few

e.g. quasar $\quad z \sim 3.78$

$\beta \sim 0.91c$!

Example $\lambda_0 = 656$ nm Balmer series

$\lambda = 1458$ nm

What is v? approaching / receding? $z = ?$

① receding since $f \sim \frac{1}{\lambda}$ is redshift (smaller)

② $$f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0$$

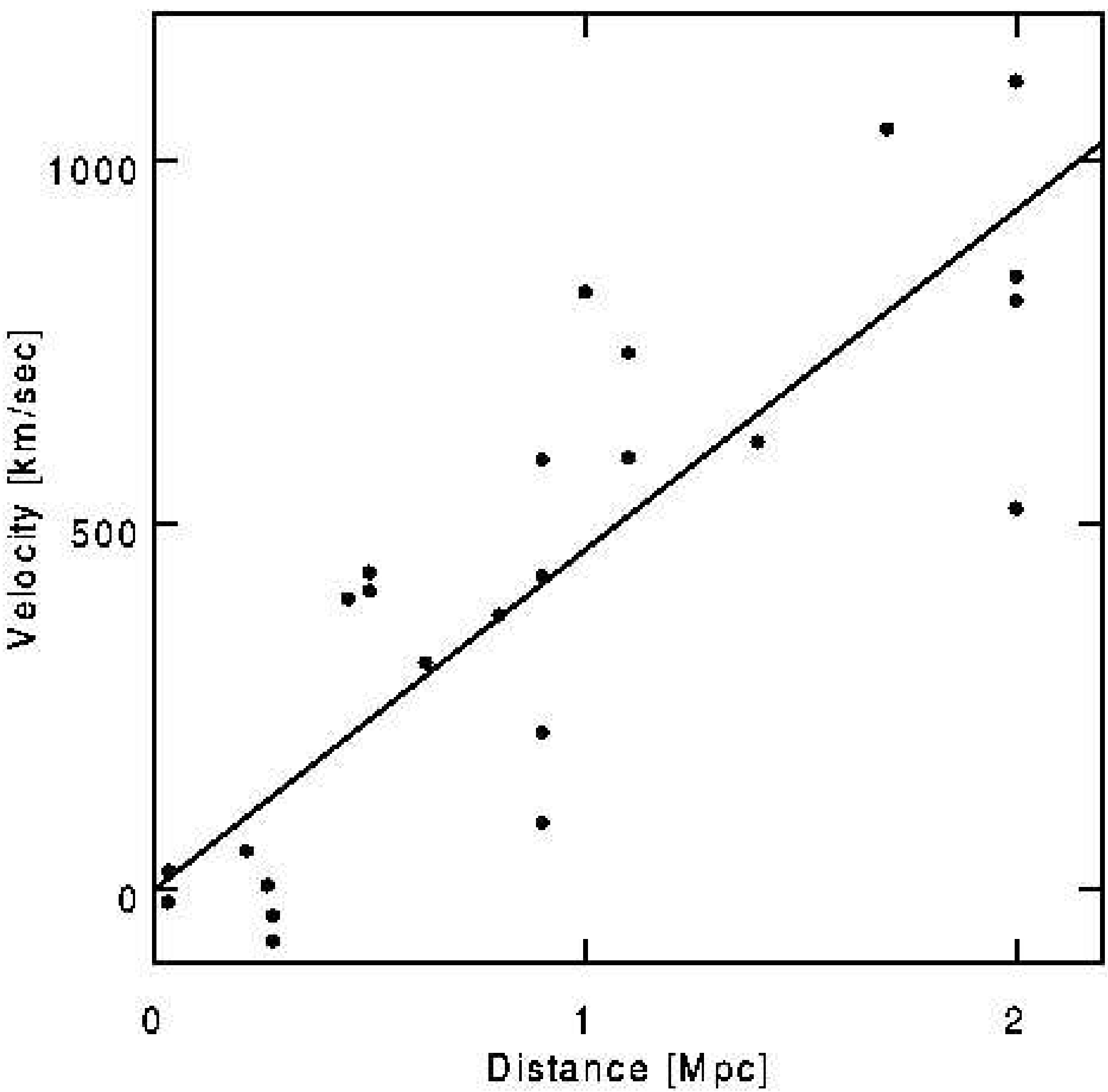
$$\frac{1-\beta}{1+\beta} \sim \left(\frac{f}{f_0}\right)^2 \sim \left(\frac{\lambda_0}{\lambda}\right)^2 \sim \left(\frac{656}{1458}\right)^2 \sim 0.202$$

$$\beta \sim 0.664$$

③ $$z = \frac{f_0 - f}{f} = \frac{\frac{c}{\lambda_0} - \frac{c}{\lambda}}{\frac{c}{\lambda}}$$

$$= \frac{\lambda - \lambda_0}{\lambda_0} = \frac{1458-656}{656}$$

$$\sim 1.22$$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1
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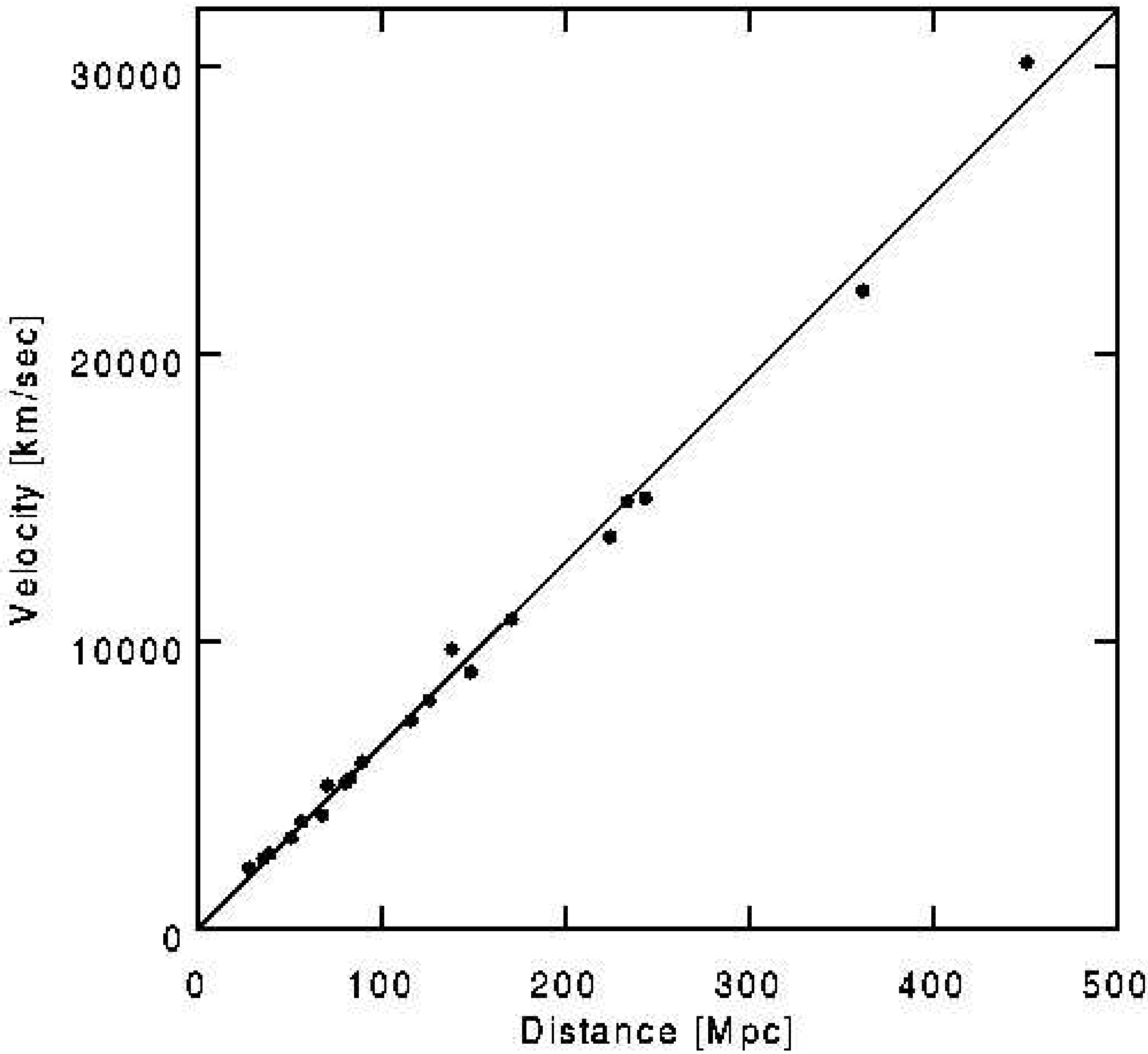
Distance [Mpc]



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Velocity [km/sec]
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400
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Distance [Mpc]