PHYSICS 247 Lecture 19

Gary Shiu

10/18/06

Today: Paradox, paradox, ...

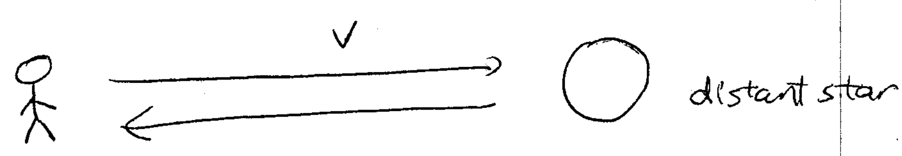
Paradoxes in Relativity

Different observers seem to have equally valid but apparently inconsistent result because $\beta \sim 1$ is very different from our intuition at $\beta \sim 0$.

Most famous example [more in lab session]

Twin Paradox

Two people were born in the same year.
Let's call them Ashton & Demi

Ashton 

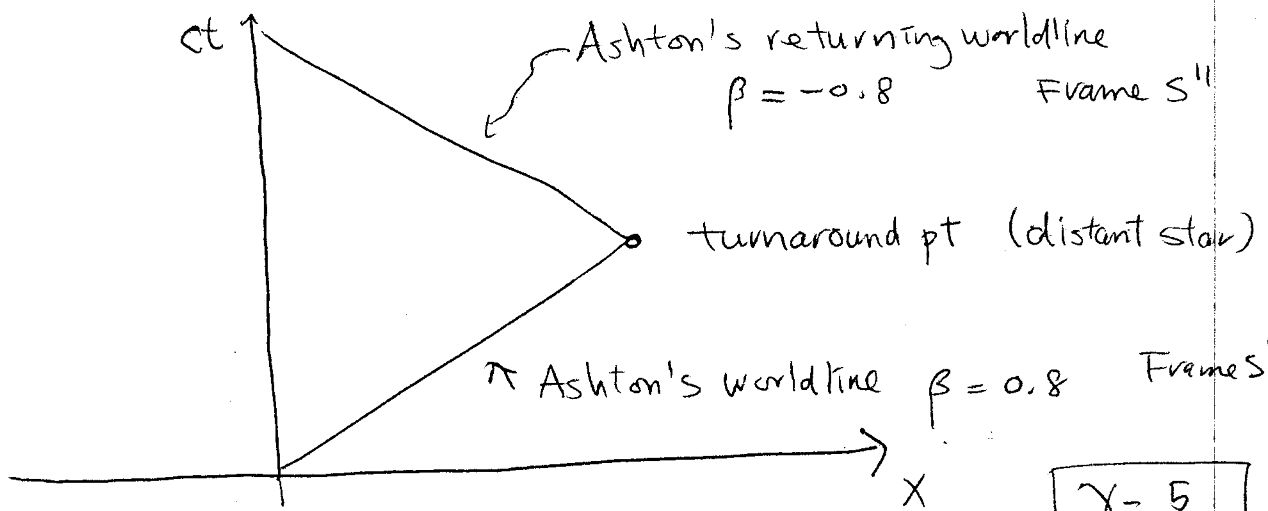
Demi  stays on earth, making movies

When Ashton returns home, he finds Demi aged compared to himself due to time dilation

A paradox : motion is relative, each can regard the other as traveler, each will find the other to be younger

$\Rightarrow$ seemingly a contradiction

Let's be more specific



$\gamma = \frac{5}{3}$

From Demi's frame: Ashton coasts in S' for a time interval $\Delta t = 5$ years and in S'' for an equal time

$\Rightarrow$ Demi is 10 years older when Ashton returns

However, Ashton measures the proper time

$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{5 \text{ years}}{\frac{5}{3}} = 3 \text{ years}$$

and same time for return

$\Rightarrow$ Ashton is 6 years older

$\downarrow$

4 years younger than Demi

<u>Dilemma</u> : From Ashton's pt of view, Demi is moving at $V = -0.8c$ & $+0.8c$ respectively in the time intervals

$\Rightarrow$ appears that Demi only aged

$$\frac{3}{\gamma} \times 2 = \frac{9}{5} \times 2 = 3.6 \text{ years}$$

<u>Reason</u> : Not Symmetrical

Demi stays in a <u>single</u> inertial frame

Ashton changes inertial frames $S' \rightarrow S''$

Although it takes a small fraction of time, it is absolutely crucial for one of them to change frames in order to compare times.

To convince Ashton : <u>spacetime interval</u> (invariant)

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

in both Ashton & Demi's frames

<u>Ashton</u> : $\Delta t = \Delta \tau$ proper time

$\Delta x = 0$

$$(\Delta s)^2 = (3 \text{ years} \cdot c)^2 = 3^2 \text{ (light years)}^2$$

<u>Demi</u> $\Delta x = v \Delta t = \beta c \Delta t$

$$\Rightarrow (\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2$$

$$= (5 \text{years} \cdot c)^2 - (0.8 \cdot c \cdot 5 \text{years})^2$$

$$= (5^2 - 4^2)(\text{light-years})^2$$

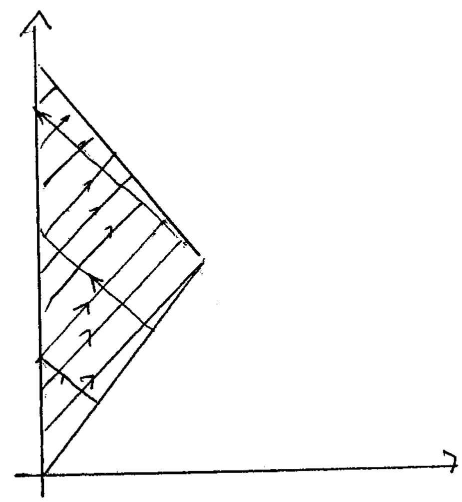
$$= 3^2 (\text{light-years})^2$$

<u>Special relativity</u> works for inertial frames

Ashton : non-inertial frame → falls outside SR

To formulate physics in a way invariant under accelerated observer → general relativity

To clarify the paradox : Ashton decides to go on another trip. This time Ashton & Demi agrees to send each other a light signal every year.



Last signal sent when they reunite

Demi sends 10 flashes

Ashton " 6 "

Although each sends a signal / year, they don't receive signal at that frequency

Why ?

Doppler's effect at work !

First half of the trip

$$\frac{f}{f_0} = \sqrt{\frac{1-\beta}{1+\beta}} = \sqrt{\frac{1-0.8}{1+0.8}} = \frac{1}{3} \quad \text{(receding)}$$

⇒ Demi receives 3 flashes in 1st 9 years

Ashton " 1 flash " " 3 years

Second half of the trip

$$\frac{f}{f_0} = \sqrt{\frac{1+\beta}{1-\beta}} = 3 \quad \text{(approaching)}$$

⇒ Demi receives 3 flashes in 1 year

Ashton " 9 " in 3 years

⇒ Demi receives 6 flashes in total

Ashton " 10 " " "