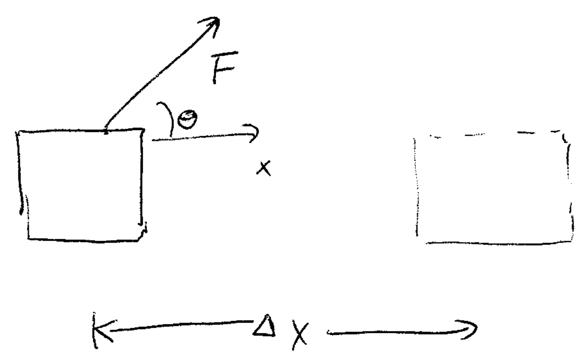
PHY 247 Lecture 20 Gary Shiu 10/23/06

T&M Chapter 6 Work and Energy

A lot of calculations in mechanics that we studied in previous chapters can be simplified using the concepts of work and energy and their conservation laws (Chapter 7)

Definition

$$\Delta W = F \cos\theta \, \Delta x$$



θ = angle btn force and displacement

Units: $\boxed{1\,J = 1\,Nm}$

$\Delta W > 0$ if $\vec{F}$ and displacement have same directions

< 0 " " " " " " " " opposite directions

More generally, we can write dW in terms of dot product:

$$dW = \vec{F} \cdot d\vec{s}$$

Dot product projects $\vec{F}$ along $d\vec{s}$

or 〃 $d\vec{s}$ along $\vec{F}$

To check whether we understand dot product

Consider $\frac{d}{dt}(\vec{v} \cdot \vec{v}) = \vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v}$

$$= 2\,\vec{a} \cdot \vec{v}$$

From Newton's law

$$\vec{F} = m\vec{a}$$

$$\Rightarrow \quad dW = m\,\vec{a} \cdot d\vec{s}$$

$$= m\,\vec{a} \cdot \vec{v}\,dt$$

$$= \frac{m}{2}\frac{d}{dt}(\vec{v} \cdot \vec{v}) \quad dt$$

$$= \frac{d}{dt}\Big(\underbrace{\frac{m}{2}v^2}_{\text{Do we recognize this?}}\Big) \quad dt$$

Work - Energy Theorem

$$\boxed{W_{total} = \Delta K}$$

Will generalize this to systems that cannot be modeled by a particle

$$dW = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)dt = \frac{dK}{dt}\,dt = dK$$

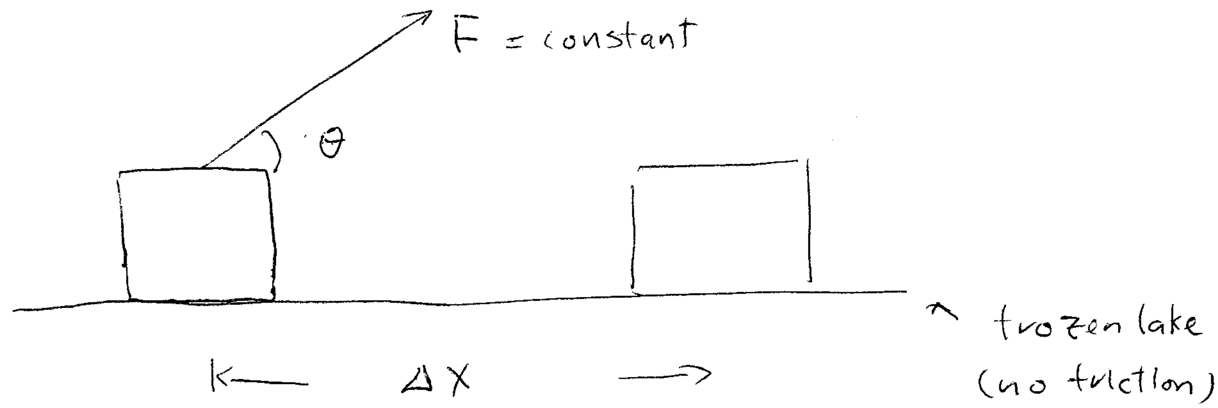
$$W_{total} = \int_1^2 \vec{F}\cdot d\vec{s} = \int_1^2 dK = K_2 - K_1$$

Power = rate of work done

$$\boxed{P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}} = \frac{dK}{dt}$$

unit of power is 1 Watt = 1 J/s

Example



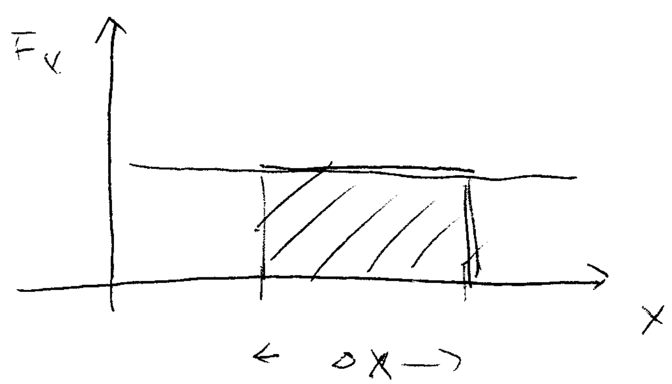
$$W = F \, \Delta X \cos\theta$$

Work-energy theorem

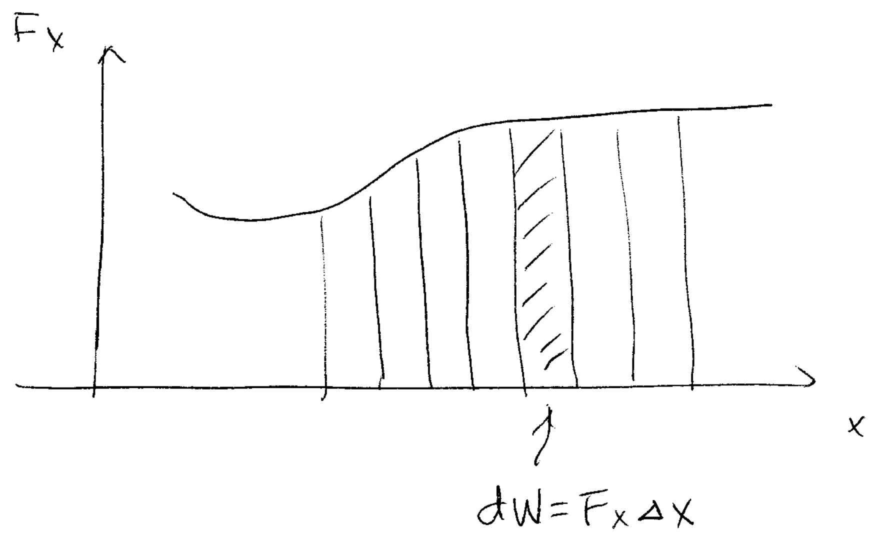
$$\frac{1}{2}mv_f^2 - \underbrace{\frac{1}{2}mv_i^2}_{=0} = W_{total} = F \, \Delta x \cos\theta$$

$$v_f = \sqrt{\frac{2F\,\Delta x\cos\theta}{m}}$$

Note: constant force, so easy to calculate W_{total}

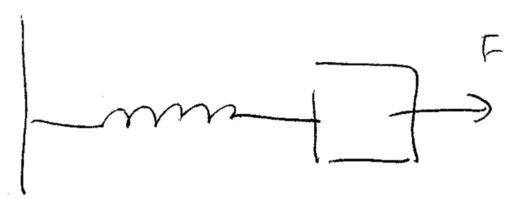


In general, $F \neq$ constant (get tired as we move the box across the lake)



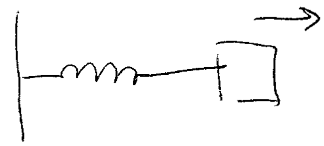
$$W = \int_{x_1}^{x_2} F_x \, dx = \text{area under } F_x \text{ versus } x \text{ curve}$$

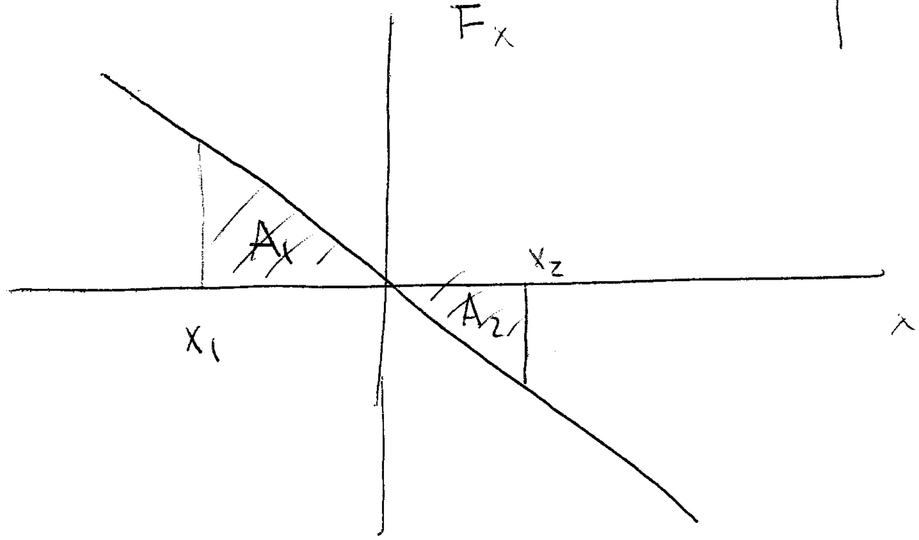
An example of a non-constant force is



$\vec{F} = -k\vec{x}$ Hooke's law

Example



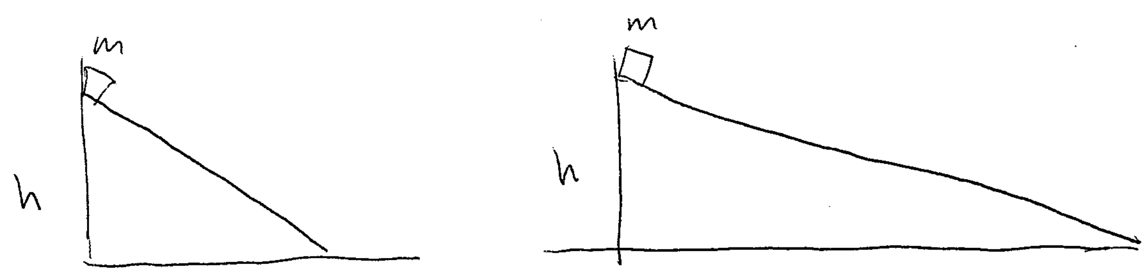


Work done by the spring on block when we move it from x_1 to x_2

$$W = \int_{x_1}^{x_2} F_x \, dx = \text{area under curve}$$

$$= A_1 - A_2$$

Example



What is the final velocity of m ?

$$mgh = \frac{1}{2} m v_f^2 \quad \Rightarrow \quad v_f = \sqrt{2gh}$$

independent of m

independent of slope

Exercise : show this by Newton's law

Q : Do they arrive the bottom of slope at the same time ? [No]

Potential Energy

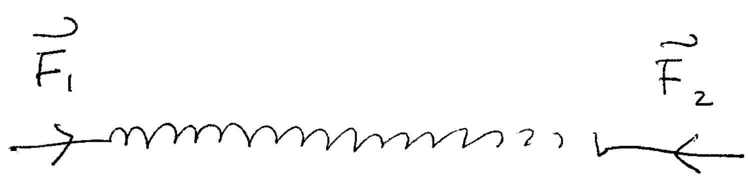
No change in kinetic energy, only potential energy

Suppose we take the box as our system,

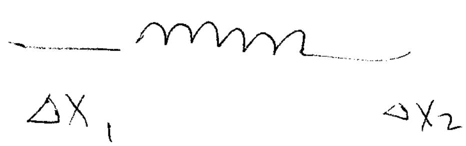
external force lifting the box $= mg$ pointing up

$\Rightarrow$ $W = mgh$ = change in potential energy

Another example

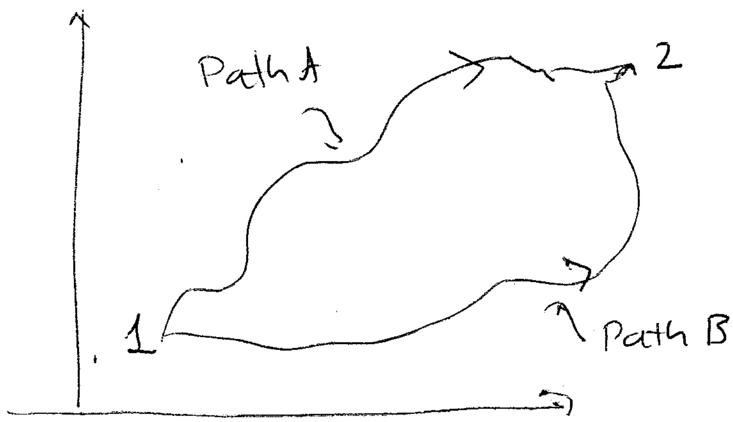


$\vec{F}_1, \vec{F}_2$ equal and opposite



Again, work done = change in elastic potential energy

Conservative Forces



Work done independent of path : conservative

e.g. gravitational force, elastic force

Useful because we can define $U(\vec{r})$ for such forces

Potential Energy function

$$W = \int_1^2 \vec{F} \cdot d\vec{s} = -\Delta U = -(U_2 - U_1)$$

$$dU = -\vec{F} \cdot d\vec{s}$$

For gravity $\quad dU = mg\,dy$

$$\Rightarrow U = U_0 + mgy$$

U_0 ← constant

Elastic Potential Energy of a spring

$$dU = -(-kx\,dx) = kx\,dx$$

$$\boxed{U = \frac{1}{2}kx^2}$$

Non-conservative forces

e.g. ① friction

② $\vec{F} = F_0\hat{\phi}$

$$\oint_{\text{closed path}} \vec{F}\cdot d\vec{s} = F_0 2\pi r \neq 0$$

$\Rightarrow$ not conservative

Potential Energy & Equilibrium

$$du = -F_x \, dx$$

$$F_x = -\frac{du}{dx}$$

In 3D:

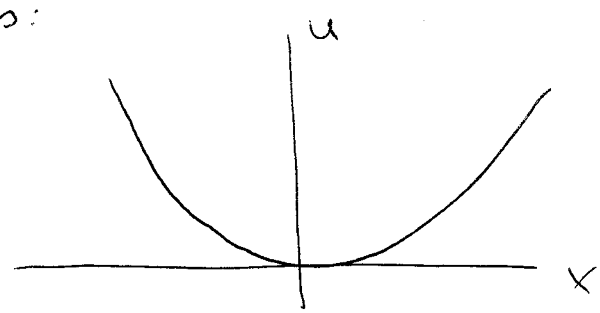
$$F_x = -\frac{du}{dx}$$

$$F_y = -\frac{du}{dy}$$

$$F_z = -\frac{du}{dz}$$

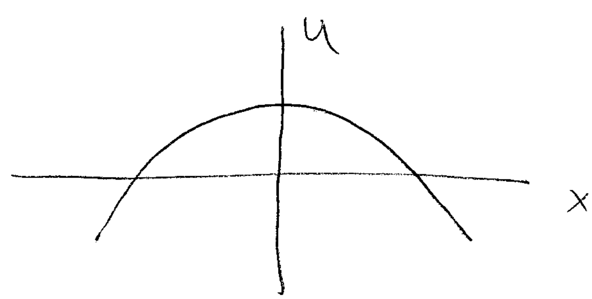
Equilibrium when $F = 0 \Rightarrow \frac{du}{dx} = 0$

Three cases:



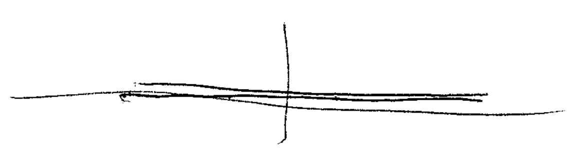
stable

$$\frac{d^2u}{dx^2} > 0$$



unstable

$$\frac{d^2u}{dx^2} < 0$$



neutral

$$\frac{d^2u}{dx^2} = 0$$