PHY 247 Lecture 21 Gary Shiu 10/25/06

Chapter 7 Conservation of Energy

We will only cover sections 7-1, 7-2

The topics in sections 7-3 and 7-4 will be covered later in the course.

In a system of particles, the work energy theorem discussed earlier will be generalized to

$$\boxed{W_{ext} = \Delta E_{sys} = \Delta E_{mech} + \Delta E_{thermal} + \Delta E_{chem} + \cdots}$$

Conservation of energy simplifies calculations greatly

Why? Instead of vectors (Newton's law) we are dealing with scalars (potential, ...)

Consider only mechanical forces for now

$$W_{total} = \sum_i W_i = \sum_i \Delta K_i = \Delta K_{system}$$

Sum over objects in the system

$$W_{total} = W_{ext} + W_{nc} + W_c$$

non-conservative conservative

But $W_c = -\Delta U_{sys}$

$$\Rightarrow \quad W_{ext} + W_{nc} = \Delta K_{sys} + \Delta U_{sys} \equiv \Delta E_{mech}$$

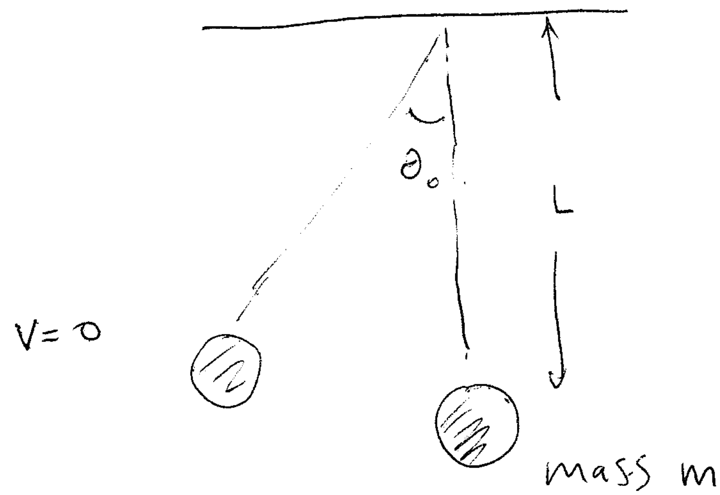
mechanical energy

If total work done by external forces and by all internal nonconservative forces is zero

$$\Rightarrow \quad E_{mech} = K_{sys} + U_{sys} = \text{constant}$$

or $\boxed{K_f + U_f = K_i + U_i}$, Conservation of mechanical energy

Example



- Find v at bottom.
- What is T at bottom

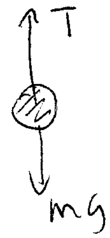
Conservation of energy

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

$$\frac{1}{2}mv_f^2 + 0 = 0 + mgh$$

$$v_f = \sqrt{2gh} = \sqrt{2gL(1-\cos\theta_0)}$$

To find T, draw force diagram



$$T - mg = ma = m\frac{v_f^2}{L}$$

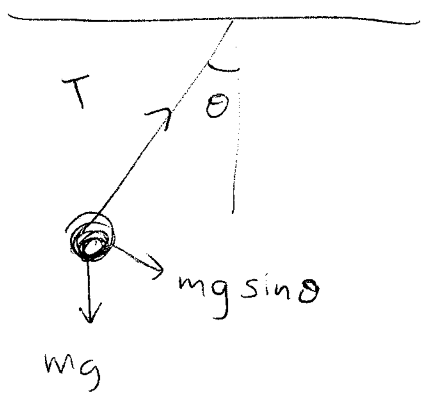
$$T = mg + 2mg(1-\cos\theta_0)$$

$$= (3 - 2\cos\theta_0)mg$$

Check ① $T \geq mg$ because of centripetal acceleration

② $T = mg$ when $\theta_0 = 0$

Solve problem using Newton's law



$$mg\sin\theta = -m\frac{d^2}{dt^2}(L\theta) = -mL\ddot{\theta}$$

$$\ddot{\theta} = -\frac{g}{L}\sin\theta$$

For small θ

$$\ddot{\theta} = -\frac{g}{L}\theta$$

$$\theta = A\cos\sqrt{\frac{g}{L}}\,t + B\sin\sqrt{\frac{g}{L}}\,t$$

Initial conditions: $\left.\begin{array}{l}\theta = \theta_0 \\ \dot{\theta} = 0\end{array}\right\}$ at $t=0$

$$\Rightarrow \quad \theta_0 = A$$

$$0 = B\sqrt{\frac{g}{L}}$$

$$\Rightarrow \quad \theta = \theta_0 \cos\sqrt{\frac{g}{L}}\, t$$

The velocity at bottom of arc

$$V = L|\dot{\theta}| = +L\,\theta_0\sqrt{\frac{g}{L}}\,\sin\sqrt{\frac{g}{L}}\,t$$

$$= +\sqrt{gL}\,\theta_0\,\sin\sqrt{\frac{g}{L}}\,t$$

$$= +\sqrt{gL}\,\theta_0 \qquad \text{since } \theta = 0 \text{ at bottom}$$

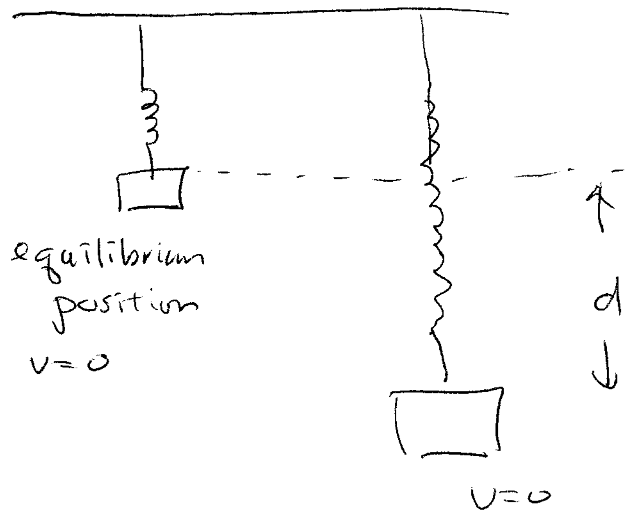
Check $\quad V = \sqrt{2gL(1-\cos\theta_0)}$

$$\simeq \sqrt{2gL\left(1 - 1 + \frac{\theta_0^2}{2} + \cdots\right)}$$

$$= \sqrt{gL}\,\theta_0$$

Note: We can solve $\theta(t)$ without making small θ approx but the answer is expressed in terms of special functions

Example



Conservation of energy

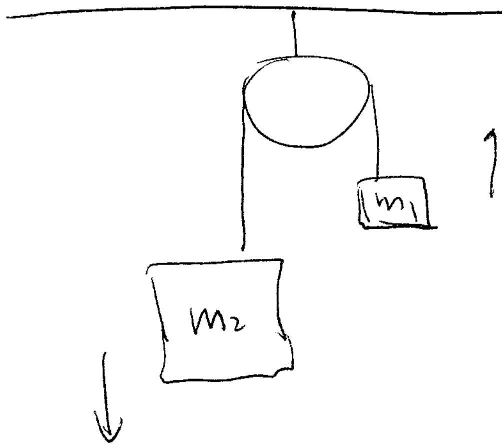
$$mgy_f + \frac{1}{2}ky_f^2 + \frac{1}{2}mv_f^2 = mgy_i + \frac{1}{2}ky_i^2 + \frac{1}{2}mv_i^2$$

$$mg(-d) + \frac{1}{2}k(-d)^2 + 0 = 0 + 0 + 0$$

$$\boxed{d = \frac{2mg}{k}}$$

Gravitational → elastic

Example Atwood's machine



$m_2 > m_1$

initially at rest

Conservation of energy

$$\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + m_1 g h - m_2 g h = 0$$

$$v = \sqrt{\frac{2(m_2 - m_1) g h}{m_1 + m_2}}$$

Since we have kept h arbitrary

$$a = \frac{dv}{dt} = \frac{dv}{dh}\frac{dh}{dt} = v\frac{dv}{dh}$$

$$= \sqrt{\frac{2(m_2-m_1)gh}{m_1+m_2}} \cdot \sqrt{\frac{2(m_2-m_1)g}{m_1+m_2}} \frac{1}{2\sqrt{h}}$$

$$= \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

Solve a, v by Newton's law

$$m_2 g - T = m_2 a \qquad\qquad T - m_1 g = m_1 a$$

Adding these eqns

$$(m_2 - m_1) g = (m_1 + m_2) a$$

$$\Rightarrow \quad a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

Constant acceleration

$$\Rightarrow \quad v^2 = 2ah$$

$$\Rightarrow \quad v = \sqrt{\frac{2(m_2 - m_1)}{m_1 + m_2} gh}$$