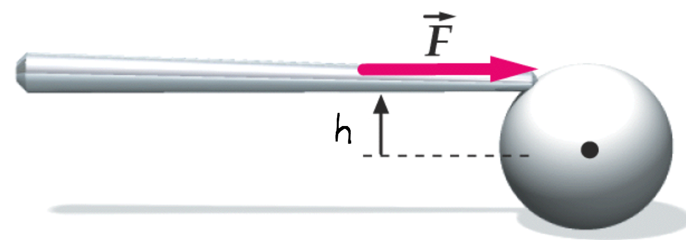
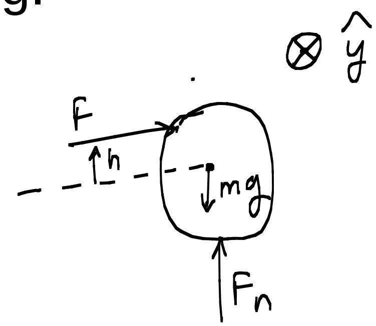
Lecture 29

Note Title 11/12/2006

Playing Pool (pg. 289)



A cue stick hits a cue ball horizontally a distance h above the center of the ball. Find the value of h for which a cue ball will roll without slipping from the beginning.



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \hat{y}\, Fh = \alpha I \,\hat{y}$$

$$F = ma$$

$$a = R\alpha \Rightarrow \alpha = \frac{a}{R}$$

$$\therefore\ Fh = \frac{a}{R} I$$

$$= \frac{F}{mR} I$$

$$\Longrightarrow h = \frac{I}{mR} = \frac{\frac{2}{5}\cancel{m}R^{\cancel{2}}}{\cancel{m}\cancel{R}} = \boxed{\frac{2}{5}R}$$

Hit above $\frac{2}{5}R$, torque will be larger $\Rightarrow$ slip since α is larger

Hit below $\frac{2}{5}R$, " " " smaller $\Rightarrow$ slip since α is smaller

Example

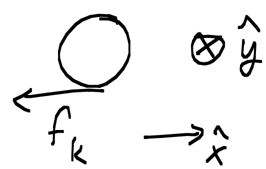
A Billiard ball of mass M and radius R is given a sharp blow by a cue stick. The applied force is horizontal and passes through the center of the ball. The initial velocity of the ball is v_0. The coeff. of friction is μ_k.

For how many seconds does the ball slide before it begins to roll w.o. slipping?

ANS

Since hit through the center of the ball, $\vec{r} \times \vec{F} = 0$.

$\Rightarrow$ No rotation just after the blow.



$$\vec{f}_k = -\hat{x}\,\mu_k M g \Rightarrow \vec{a}_{cM} = -\hat{x}\,\mu_k g$$

$$\vec{\tau} = \vec{r} \times \vec{f}_k = R \mu_k M g\,\hat{y} = I \alpha\,\hat{y}$$

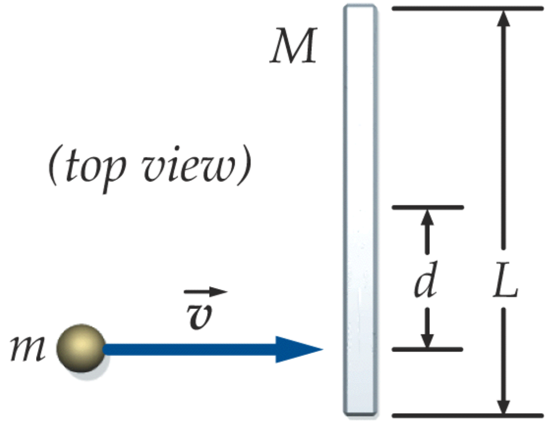
Our desired condition is $v_{cM} = R\omega$

$$v_{cM} = v_0 - \mu_k g \Delta t$$

$$\omega = \frac{R \mu_k M g}{I} \Delta t$$

$$v_0 - \mu_k g \Delta t = \frac{R^2 \mu_k M g \Delta t}{I}$$

$$\therefore \quad v_0 = (\Delta t)\left(\frac{\cancel{R}^2 \mu_k \cancel{M} g}{\frac{2}{5} \cancel{M} \cancel{R^2}} + \mu_k g \right) \Rightarrow \Delta t \frac{7}{2} \mu_k g = v_0 \Rightarrow \boxed{\Delta t = \frac{2 v_0}{7 \mu_k g}}$$

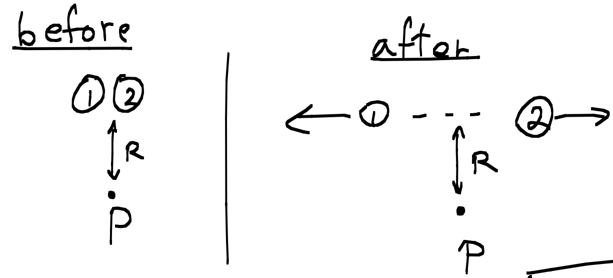


Shown is the situation before the collision. What happens after the collision if it is elastic?

Example

Suppose the force law between two particles is $\vec{F}_{21} = f(|\vec{r}_1 - \vec{r}_2|)\dfrac{[\vec{r}_2 - \vec{r}_1]}{|\vec{r}_2 - \vec{r}_1|}$ (force on 2 exerted by 1)

What is the final total angular momentum about the point P?



ANS

Angular momentum conservation says $\boxed{\vec{L}_f = 0}$. However, consider from the point of view of the theorem concerning how internal torques cancel.

$$\vec{\tau}_{21} = \vec{r}_2 \times \vec{F}_{21} = f(|\vec{r}_1 - \vec{r}_2|)\, \vec{r}_2 \times \frac{[\vec{r}_2 - \vec{r}_1]}{|\vec{r}_2 - \vec{r}_1|} = f(|\vec{r}_1 - \vec{r}_2|)\frac{(-\vec{r}_2 \times \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{\tau}_{12} = \vec{r}_1 \times \vec{F}_{12} = f(|\vec{r}_2 - \vec{r}_1|)\, \vec{r}_1 \times \frac{[\vec{r}_1 - \vec{r}_2]}{|\vec{r}_1 - \vec{r}_2|} = f(|\vec{r}_2 - \vec{r}_1|)\frac{(-\vec{r}_1 \times \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

Since $\vec{r}_1 \times \vec{r}_2 = -\vec{r}_2 \times \vec{r}_1$,

$$\vec{\tau}_{21} + \vec{\tau}_{12} = 0 \Rightarrow$$

Internal torques cancel. Angular momentum is conserved