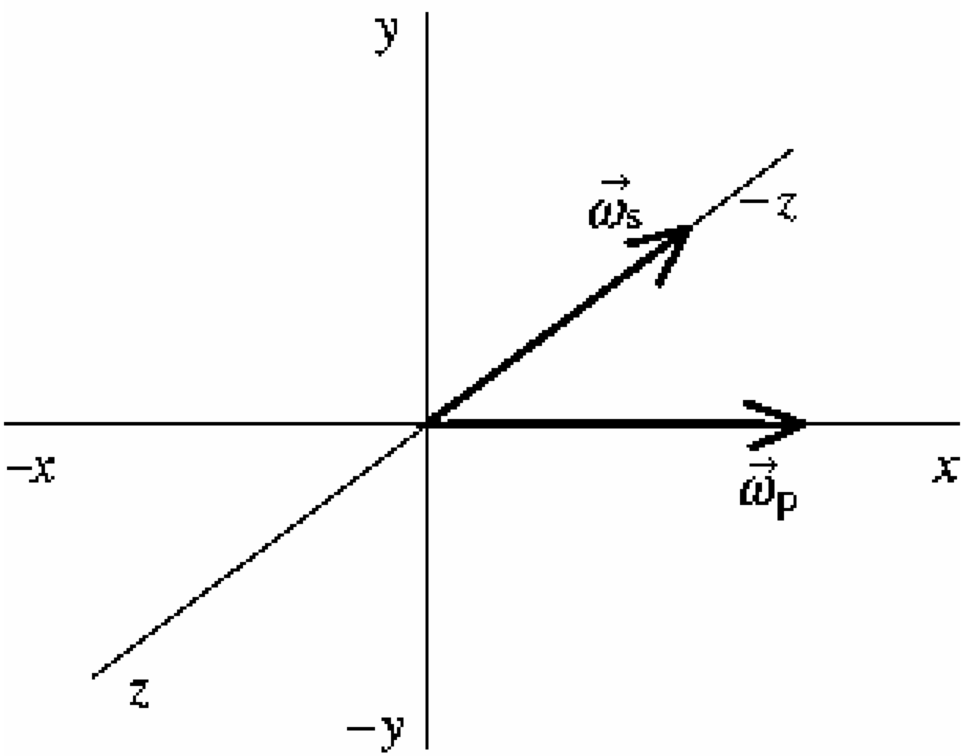
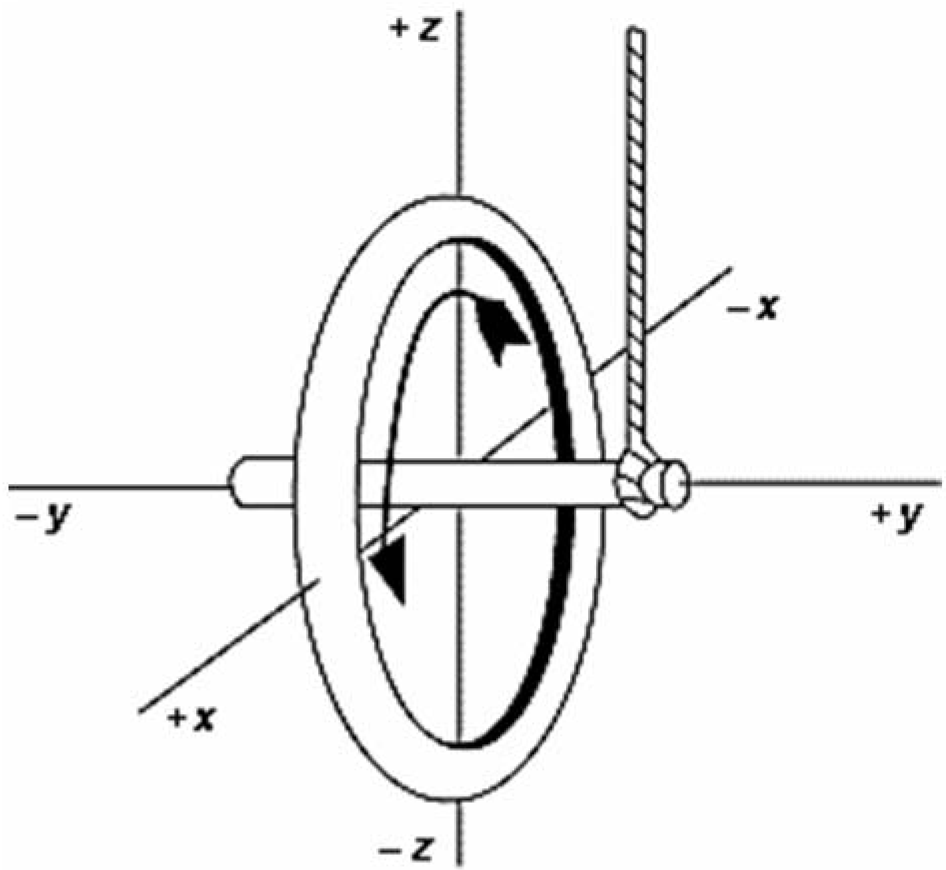
Lecture 30



The figure shows vectors representing the angular velocity of precession ω_p and the spin velocity ω_s. The associated torque vector points along which of the axes?
A) $-x$ B) y C) z D) $-z$ E) None of these is correct.



A wheel is set spinning and then is hung by a rope placed at one end of the axle. The precession vector of the spinning wheel points in the direction of

A) z B) $-y$ C) $-z$ D) $-x$ E) y

If an object were to suddenly shrink and decrease its moment of inertia by a factor of 3, what is the difference in energy between the final and initial rotational kinetic energies?

A) 3 times initial K_{rot}

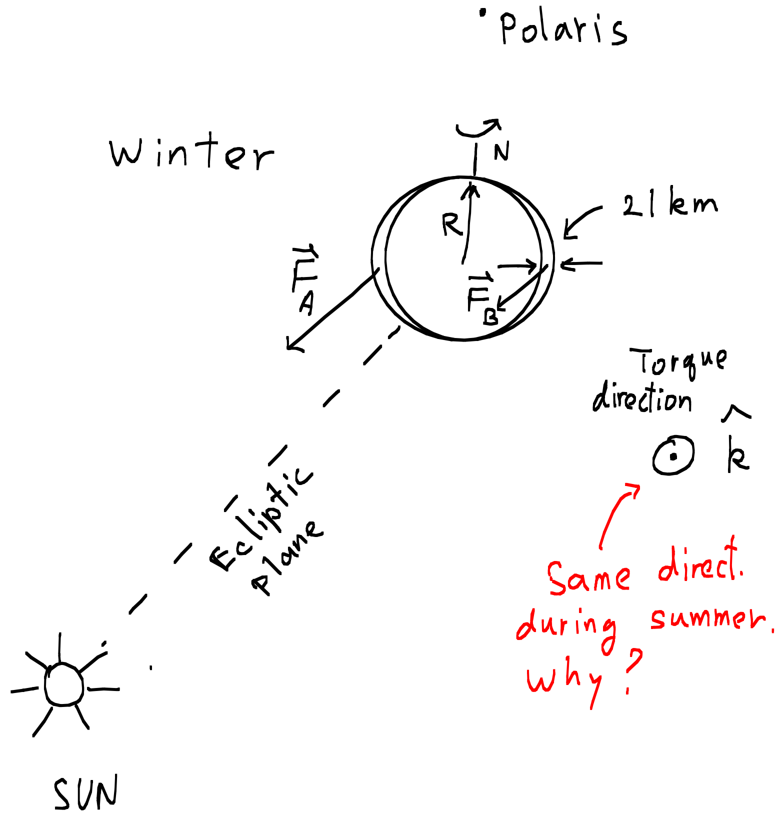
B) 9 times initial K_{rot}

C) 8 times initial K_{rot}

D) 2 times initial K_{rot}

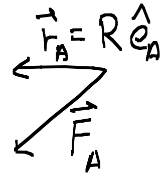
E) zero, K_{rot} remains the same

Precession of Equinoxes

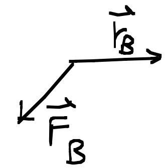


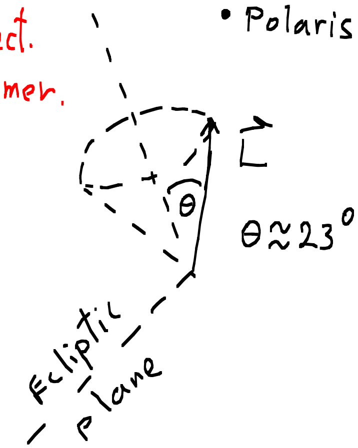
$R \approx 6.4 \times 10^3$ km

$|\vec{F}_A| > |\vec{F}_B|$



$|\vec{r}_A \times \vec{F}_A| > |\vec{r}_B \times \vec{F}_B|$

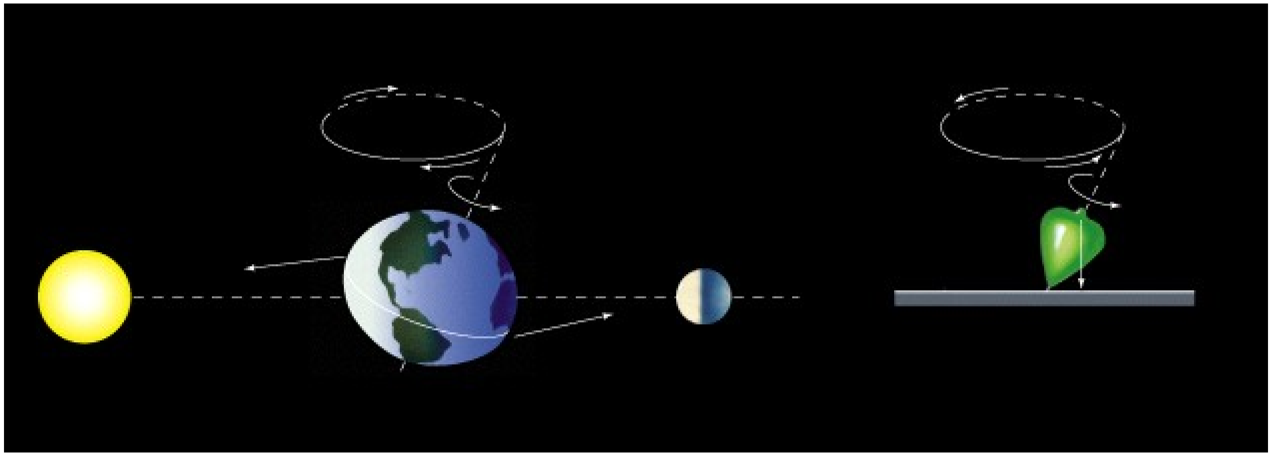




In a "similar" fashion, the moon exerts an average torque on earth. The moon's torque is about 2X that of the sun.

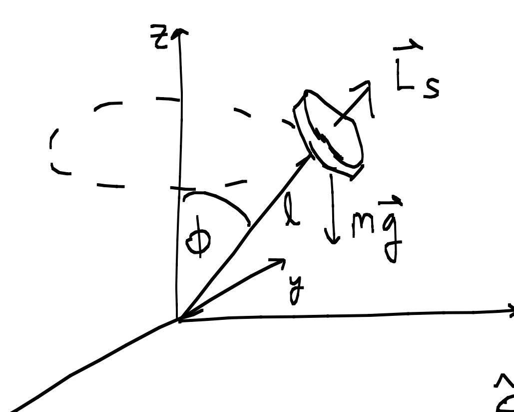
The precession period is around 26,000 years!

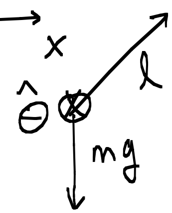
The spring equinox occurs at the instant the sun is directly over the equator in its apparent passage from north to south. Due to the precession of the earth's axis the position of the sun against the backdrop of stars shifts by 50" of arc each year.



Example

Gyroscope precession





$$\vec{r} \times (m\vec{g}) = \frac{d\vec{L}_s}{dt}$$

$$|\vec{r}| = l$$

$$\therefore \left|\frac{d\vec{L}_s}{dt}\right| = l\, m\, g \sin\phi$$

direction is $\perp$ to $\vec{L}_s$.

$\Rightarrow$ $\frac{dL_s}{dt} = 0$ while $\frac{d\vec{L}_s}{dt} \neq 0$

i.e. $\frac{d\vec{L}_s}{dt} = L_s \frac{d}{dt}\left(\cos\phi\, \hat{z} + \sin\phi\, \hat{r}\right)$ in cylindrical coords.

$$= L_s\, l \sin\phi \frac{d\hat{r}}{dt} = L_s\, l \sin\phi\, \omega\, \hat{\theta}$$

Precession frequency

$$\Rightarrow l\, m\, g\, \cancel{\sin\phi} = L_s\, (\cancel{\sin\phi})\, \omega \Rightarrow$$

$$\boxed{\omega = \frac{mgl}{L_s}}$$

For point particle,

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$K = \frac{1}{2} m \vec{v}^2$$

For extended systems

$$\vec{L}_{sys} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\frac{d\vec{L}_{sys}}{dt} = \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{\tau}_i$$

$$K = \sum_i \frac{1}{2} m_i \vec{v}_i^{\,2} = \sum_i \frac{1}{2} \frac{|\vec{L}_i|^2}{I_i}$$

For a fixed axis,

$$\vec{L}_i = I_i \vec{\omega}_i$$

Conservation of angular momentum follows from total external torque $= 0$ and assumptions about internal forces. Newton's 3rd law is not enough.

Conservation of linear momentum follows from total external force $= 0$. Newton's 3rd law is enough.