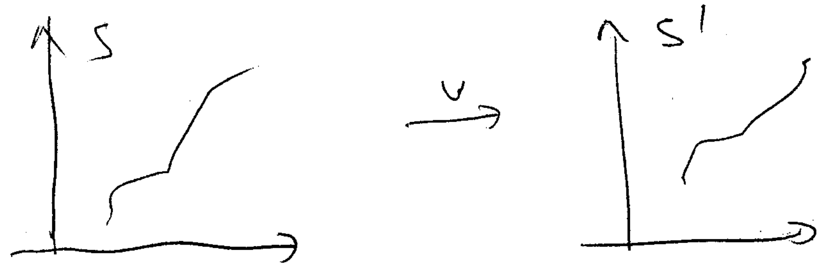
Relativity Lecture Ch 2 (Lecture 31) 11/20/06

A few weeks ago, Kinematics : Lorentz transf



But we don't know yet how to describe motion

→ Dynamics.

Analogy: we have understood the rules of describing projectile motion in different reference frames,

but we haven't figured out how to calculate the projectile motion!

In classical mechanics we need Newton's law

$$\vec{F} = m\vec{a}$$

For relativistic motion, $\vec{F} = m\vec{a}$ is no longer true.

Why?

(1) F = constant $t_0 \Rightarrow$ $V \to \infty$

but $V \leq c$ speed limit

(2) $F = 0$ $\Rightarrow$ Conservation of momentum

$$\sum_i m_i \vec{v}_i = \text{constant}$$

- For Galilean transformation $\vec{v}_i{}' = \vec{v}_i - \vec{V}$

  If momentum conserved in one frame, conserved in all reference frame

- Lorentz transf, velocities do not simply add

  If $\vec{p} = m\vec{v}$, then $\vec{p}$ conserved in one frame does not imply $\vec{p}$ conserved in another frame.

<u>Hallmark of relativity</u>: physics does not depend on ref. frame

↳ Relativistic momentum $\vec{p} \neq m\vec{v}$

Requirement

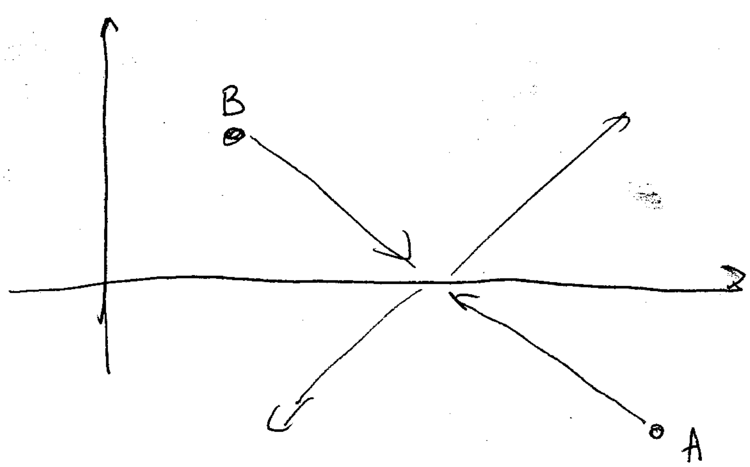
1. Momentum conservation $\Rightarrow \vec{p}$ conserved in collision in all reference frame

2. $\vec{p} \to m\vec{v}$ for $v \ll c$

$\boxed{\text{Guess}}$ $$\vec{p} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}}$$

Now, let's do a thought expt to ~~see~~ why this is true.

Imagine an elastic collision in ref frame S'



A, B identical mass

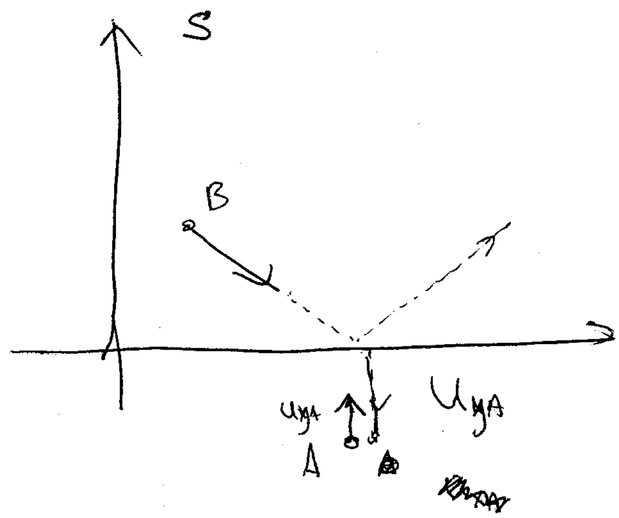
equal & opposite velocities

Total initial momentum $= 0$

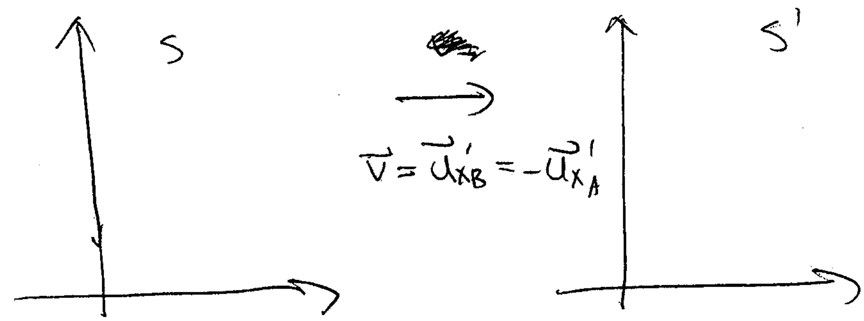
Elastic collision $\Rightarrow$ reverse y-component of velocity

Total final momentum $= 0$

Let's go to a reference frame S where $u_{x_A} = 0$



In other words



Had it been Galilean transformation, u_y is unaffected by transf

$\Rightarrow$ momentum conserved $\left(\Delta u_{yA} + \Delta u_{yB} = 0 \right)$

But as we have learned in Ch 1,

$$u_y' = \frac{u_y \sqrt{1-\beta^2}}{1 - u_x V/c^2}$$

Not only was u_y affected but u_{yA}' & u_{yB}' affected differently

$u_{xA} = 0 \quad u_{xB} \neq 0$

Since $u'_{yB} = u'_{yA}$ magnitude

$$\Rightarrow \quad u_{yA} = u_{yB} \frac{1}{1 - u_{xB} v/c^2}$$

Elastic collision $\Rightarrow$ $\Delta u_y = 2 u_y$

$\Rightarrow$ Momentum not conserved in frame S !

$$\boxed{\text{If Momentum is conserved in } S', \text{ not conserved in } S \text{ and vice versa}}$$

Need a new definition of momentum.

Suppose $\vec{p} = m(\vec{u})\,\vec{u}$

Then momentum is conserved if

$$m_B = m_A \frac{u_{yA}}{u_{yB}} = \frac{m_A}{1 - \underset{\substack{\uparrow \\ u'_{xB}}}{u_{xB} v}/c^2}$$

$$= \cancel{\frac{m_A}{1 - u_{xB} u'_{xB}/c^2}}$$

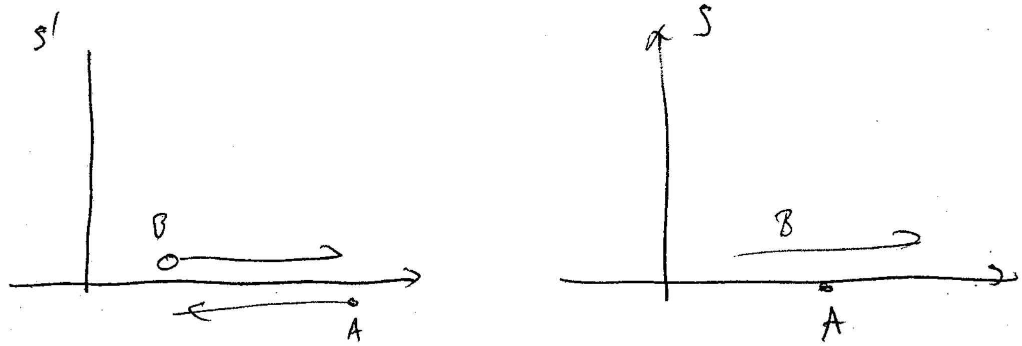
Using $u'_{xB} = \dfrac{u_{xB} - v}{1 - u_{xB} v/c^2} = v$

$$V = \frac{c^2}{u_{xB}}\left(1 - \sqrt{1 - u_{xB}^2/c^2}\right)$$

reject other root

$$\Rightarrow \quad m_B = \frac{m_A}{\sqrt{1-(u_{xB}/c)^2}}$$

Consider the special case $u_y \to 0$ <u>grazing collision</u>



$$\Rightarrow \quad \boxed{m = \frac{m_0}{\sqrt{1-u^2/c^2}}}$$

or

$$\boxed{\vec{p} = \frac{m_0\vec{u}}{\sqrt{1-u^2/c^2}}}$$

Although these eqns mean the same thing, it's better to avoid ~~definition~~ defining a <u>relativistic</u> <u>mass</u> m and a rest mass m_0