PHY 247 Lecture 32 Gary Shiu 11/22/06 (1)

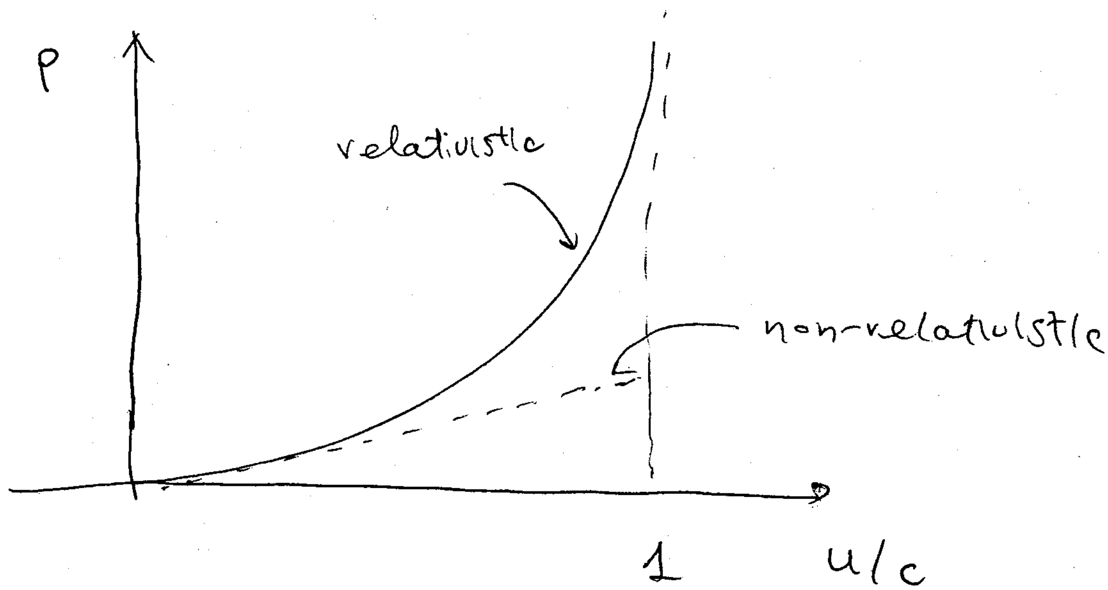
Relativistic Momentum

$$\vec{p} = \gamma m \vec{u}$$

↑ abuse of notation $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$

— Clearly $\vec{p}$ is conserved in collisions (by construction)

— $\vec{p} \sim m\vec{u}$ as $u \ll c$



Constant force $\Rightarrow$ $p \to \infty$ but $u/c \to 1$

[Note: Although derivation is done for a special case, $\vec{p} = \gamma m \vec{u}$ holds in general]

<u>Example</u> For what value of u/c will measured mass of an object γm exceed rest mass by f

$$f = \frac{\gamma m - m}{m} = \gamma - 1 = \frac{1}{\sqrt{1-u^2/c^2}} - 1$$

$$\frac{u}{c} = \frac{\sqrt{f(f+2)}}{f+1} \quad \text{Independent of } m$$

f	β
10^{-3}	0.014
10^{-2}	0.14
0.1	0.42
1	0.87
10	0.994
100	0.999

Difficult to build accelerators!

$1\,eV \sim 1.602\times10^{-19}\,C \times 1\,V = 1.602\times10^{-19}\,J$

$TeV \sim 1.602\times10^{-7}\,J$ small number

but compare with rest mass of particle

proton mass $\sim 938\,MeV \sim 938\times10^{6}\,eV \sim 10^{-27}\,kg$

Relativistic Energy

① Total energy of any isolated system is Conserved

② $E \rightarrow$ classical value when $u/c \rightarrow 0$

Newton's law $\vec{F} = m\vec{a}$

$$\Rightarrow \quad \vec{F} = \frac{d\vec{p}}{dt} \leftarrow \text{relativistic}$$

$$= \frac{d(\gamma m\vec{u})}{dt}$$

Kinetic energy = Work done in accelerating a particle from rest

$$K = \int_{u=0}^{u} \vec{F} \cdot d\vec{l}$$

Consider 1-D motion for simplicity :

$$K = \int_{u=0}^{u} F\,dx = \int_{0}^{u} \frac{d(\gamma mu)}{dt}\,dx$$

$$= \int_{0}^{u} u\,d(\gamma mu)$$

Exercise: Show $d(\gamma m u) = m\left(1-\frac{u^2}{c^2}\right)^{-3/2} du$

Proof $d(\gamma m u) = d\left(\frac{mu}{\sqrt{1-u^2/c^2}}\right)$

$$= m\left[\frac{du}{\sqrt{1-u^2/c^2}} + \frac{u^2/c^2\, du}{(1-u^2/c^2)^{3/2}}\right]$$

$$= \frac{m\, du}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\left[1-\frac{u^2}{c^2}+\frac{u^2}{c^2}\right]$$

$$= \frac{m\, du}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

Therefore

$$K = \int_0^u u \frac{m\, du}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

$$= \left.\frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}\right|_0^u = mc^2\left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}-1\right)$$

$$\boxed{K = (\gamma-1)mc^2}$$ ← Does this make sense?

For $u << c$

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} \simeq 1 + \frac{1}{2}\frac{u^2}{c^2} + \cdots$$

$$\Rightarrow \quad E = mc^2 \left[1 + \frac{1}{2}\frac{u^2}{c^2} + \cdots - 1\right]$$

$$\simeq \frac{1}{2} m u^2 \quad \checkmark$$

non-relativistic kinetic energy

<u>Interpretation</u>

$$K = \gamma mc^2 - mc^2$$

depends on speed — rest energy (always there)

Work done increases energy from $mc^2 \rightarrow \gamma mc^2$

Total energy $E = K + mc^2 = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}}$

For $u = 0$

$$\boxed{E = mc^2}$$

For $u << c$

$$E \simeq \frac{1}{2} m u^2 + mc^2$$

additive constant (ref pt)

<u>Note</u>: ① $K \to \infty$ as $u \to c$, infinite amount of work done

c plays the role of limiting velocity

② E is conserved quantity, not K ~~nor~~ or mc^2

③ E is conserved but not invariant

→ frame dependent

④ Haven't shown E is conserved yet, will do so <u>after</u> we figure out how $\vec{p}$ & E transform