Lorentz Transformation

S

$$E = \gamma m c^2$$

$$p_x = \gamma m u_x$$

$$p_y = \gamma m u_y$$

$$p_z = \gamma m u_z$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

S'

$$\gamma \rightarrow \gamma' = \frac{1}{\sqrt{1-u'^2/c^2}} \qquad \vec{u} \rightarrow \vec{u}'$$

To relate $(\vec{p}, E)$ to $(\vec{p}', E')$

First

$$\boxed{\frac{1}{\sqrt{1-u'^2/c^2}} = \gamma \frac{1 - v u_x/c^2}{\sqrt{1-u^2/c^2}}} \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

HW

$$\Rightarrow E' = \frac{mc^2}{\sqrt{1-u'^2/c^2}} = \gamma \left[\frac{mc^2}{\sqrt{1-u^2/c^2}} - \frac{mc^2 v u_x/c^2}{\sqrt{1-u^2/c^2}} \right]$$

$$= \gamma (E - v p_x)$$

$$p_x' = \frac{m u_x'}{\sqrt{1-u'^2/c^2}} = m\gamma \frac{(1-\cancel{v u_x/c^2})}{\sqrt{1-u^2/c^2}} \cdot \frac{u_x - v}{\cancel{1-\frac{u_x v}{c^2}}}$$

$$= \frac{m\gamma (u_x - v)}{\sqrt{1-u^2/c^2}}$$

$$= \gamma \left(p_x - v \frac{E}{c^2} \right)$$

$$p_y' = \frac{m u_y'}{\sqrt{1-u'^2/c^2}} = m \frac{\cancel{\gamma} (1-\cancel{v u_x/c^2})}{\sqrt{1-u^2/c^2}} \cdot \frac{u_y \cancel{\sqrt{1-v^2/c^2}}}{\cancel{1-\frac{v u_x}{c^2}}}$$

$$= \frac{m u_y}{\sqrt{1-u^2/c^2}} = p_y$$

$$p_z' = p_z$$

So

	inverse
$p_x' = \gamma (p_x - v E/c^2)$	$p_x = \gamma (p_x' + v \frac{E'}{c^2})$
$p_y' = p_y$	$p_y = p_y'$
$p_z' = p_z$	$p_z = p_z'$
$E' = \gamma (E - v p_x)$	$E = \gamma (E' + v p_x')$

Does that ring a bell?

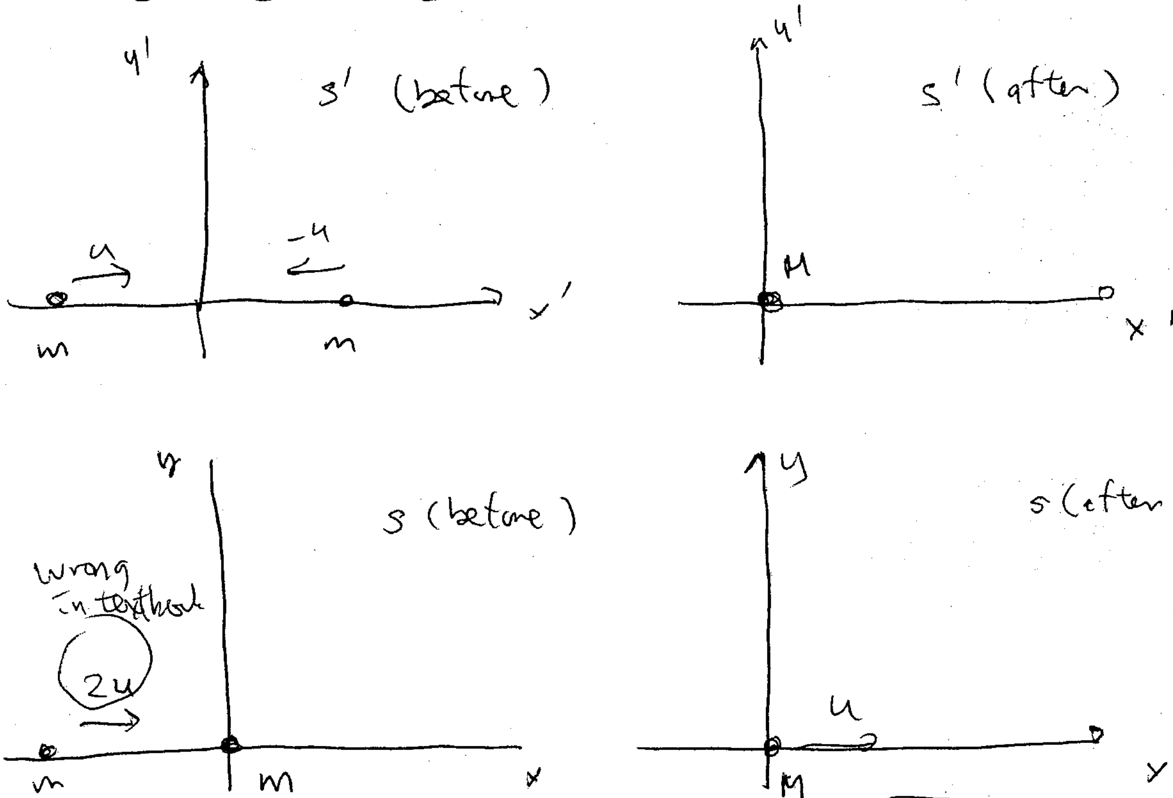
$(\vec{p}, E)$ transform like $(\vec{r}, t)$!

Conservation of Energy

Check conservation law applies to some special cases.

Elastic collision example : obviously works because only direction changes, not speed.

Inelastic collision



$\boxed{\text{Note } M \neq 2m \text{ why?}}$

Let's analyze energy conservation in the 2 reference frames respectively

Ref frame S'

Before collision: $E'_{before} = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}}$

$$= \frac{2mc^2}{\sqrt{1-u^2/c^2}}$$

After collision: $E'_{after} = Mc^2$

Note that momentum conservation $\Rightarrow M = \frac{2m}{\sqrt{1-\frac{u^2}{c^2}}}$

$\Rightarrow$ energy conserved

Ref frame S

Before collision: $E_{before} = \gamma(E'_{before} + vp_x')$

$$= \gamma\left(\frac{2mc^2}{\sqrt{1-u^2/c^2}} + 0\right)$$

After collision $E_{after} = \gamma(Mc^2 + vp_x')$

$$= \gamma\left(\frac{Mc^2}{\sqrt{1-\frac{u^2}{c^2}}} + 0\right)$$

$\Rightarrow$ energy also conserved

Though a special case, result is general

Example Cosmic Ray Muon (classic relativity example)

Rest mass $= 105.7 \frac{MeV}{c^2}$

$$u = 0.998c$$

$$E = \gamma mc^2 = \frac{1}{\sqrt{1-0.998^2}}(105.7\ MeV)$$

$$= 1670\ MeV$$

Note: Convenient unit in particle physics is eV, keV, MeV, GeV, TeV instead of kg → due to $E = Mc^2$

$$1eV = (1.602 \times 10^{-19}\ C)(1.0\ V)$$

$$= 1.602 \times 10^{-19}\ J$$

$$\text{Mass of } e^- = 9.11 \times 10^{-31}\ kg$$

$$= \frac{9.11 \times 10^{-31}\ kg \times c^2}{c^2}$$

$$= \frac{8.19 \times 10^{-14}\ J}{c^2} = \frac{0.5\ MeV}{c^2}$$

Note: LHC ~ TeV scales ⇒ doesn't mean enormous energy in terms of J

But ~~[illegible] [illegible]~~ highly relativistic for elementary particles such as p