PHY 247 Lecture 34 Gary Shiu 11/29/06 1.

Mass / Energy Conversion

$E = mc^2$ Mass & Energy are interchangable

Einstein's original paper $m = \frac{E}{c^2}$ but in the end of the paper mentioned the application of $E=mc^2$ to nuclear reaction

Binding energy : mass of a bound system is less than that of separated ~~...~~ constituents by E_b / c^2 where E_b = binding energy

$$\boxed{\sum_i m_i = M_{bound} + \frac{E_b}{c^2}}$$

Example : Deuteron ${}^2H \rightarrow p + n$

$m_p = 938.27$ MeV $/c^2$

$m_n = 939.57$ MeV $/c^2$

$m_{{}^2H} = 1875.61$ MeV $/c^2$

$\Rightarrow \quad E_b = (938.27 + 939.57 - 1875.61)$ MeV

$= 2.22$ MeV

${}^2H$ cannot spontaneously break up into p & n

2.22 MeV must be injected (e.g. energetic particle bombardment

Another example: Electron binding energy in hydrogen

$H = 1p + 1e$

$m(^1H) = m_p + m_e - \frac{E_b}{c^2}$ $\qquad E_b = 13.6\,eV$

$\frac{E_b}{\quad} << M_p + m_e$ (compared with previous example)

$\frac{E_b}{M_p c^2} \sim \frac{13.6\,eV}{938\,MeV} \sim 10^{-8}$

Such small mass difference can be measured by atomic physics. (mass spectroscopy)

Invariant Mass

When we discussed kinematics, we found a useful invariant quantity

$$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

same in all reference frames (useful in determining whether events are ~~[illegible]~~ casually connected or not – proper time etc)

What about ~~[illegible]~~ the dynamics of the system?

Hint: E & $\vec{p}$ transform in the same way as t & $\vec{x}$

$$\text{Invariant} = E^2 - p^2c^2$$

$$= \frac{m^2c^4}{1-\frac{u^2}{c^2}} - \frac{m^2u^2c^2}{1-\frac{u^2}{c^2}}$$

$$= \frac{m^2c^4\left(1-\frac{u^2}{c^2}\right)}{1-\frac{u^2}{c^2}}$$

$$= m^2c^4$$

$$\Rightarrow \boxed{(mc^2)^2 = E^2 - p^2c^2}$$

invariant = rest mass!

But is the invariant mass only invariant for a single particle?

$$E = \sum E_i$$

$$\vec{p} = \sum_i \vec{p}_i \quad \rightarrow$$

since only p_x matters

$$p_x = \sum_i p_{ix}$$

$$E^2 - p^2c^2 = \left(\sum_i E_i\right)^2 - \left(\sum_i p_{ix} c\right)^2$$

drop y & z component

$$E'^2 - p'^2c^2 = \left(\sum_i E_i'\right)^2 - \left(\sum_i p_{ix}' c\right)^2$$

$$= \left[\sum_i \gamma\left(E_i - v p_{ix}\right)\right]^2 - \left[\sum_i \gamma\left(p_{ix} - \frac{vE_i}{c^2}\right)\right]^2 c^2$$

$$= \gamma^2 \left(\sum_i E_i\right)^2 \left(1 - \frac{v^2}{c^2}\right)$$

$$- \cancel{2\gamma v \sum_i E_i \sum_j p_{jx}} + \gamma^2 v^2 \left(\sum_i p_{ix}\right)^2$$

$$- \gamma^2 c^2 \left(\sum_i p_{ix}\right)^2 + \cancel{2\gamma v \sum_i E_i \sum_j p_{jx}}$$

$$= \left(\sum_i E_i\right)^2 - c^2 \left(\sum_i p_{ix}\right)^2$$

$$= E^2 - p^2c^2$$

Useful ?

(1) One can compute rest mass without having to go to rest frame

→ useful in particle physics especially because particles are typically moving at high speed!

e.g LHC → Higgs, SUSY particles

(2) Once we know rest mass (say for electron $m = 0.511 MeV$), we can calculate dynamical variables such as momentum, speed etc

Example: Total energy of an electron produced in a particular nuclear reaction is 2.4 MeV

$$pc = \sqrt{E^2 - (mc^2)^2} = 2.34 \text{ MeV}$$

$$\Rightarrow p = 2.34 \frac{\text{MeV}}{c}$$

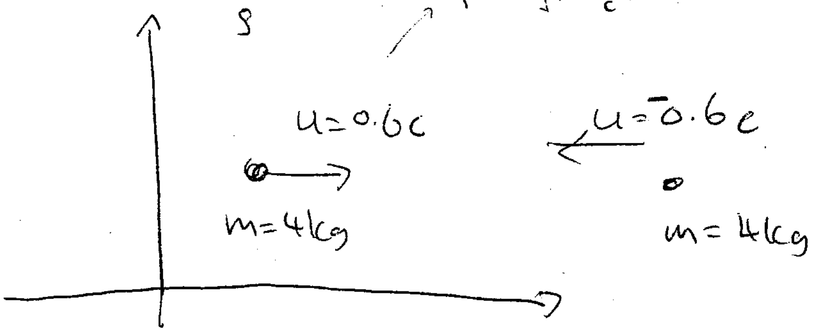
$$\Rightarrow \frac{u}{c} = \frac{p}{E} = 0.975$$

Note that invariant mass of the system is not the sum of rest mass of constituents

$$M \neq \sum_i m_i$$

Example

$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = 3c\ \text{kg}$$



$$\frac{E_{total}}{c^2} = \sqrt{m_1^2 + \left(\frac{p_1}{c}\right)^2} + \sqrt{m_2^2 + \left(\frac{p_2}{c}\right)^2}$$

$$= \left(\sqrt{4^2 + 3^2}\ \text{kg}\right) \times 2$$

$$= 10\ \text{kg}$$

$p_{total} = 0$ center of mass frame

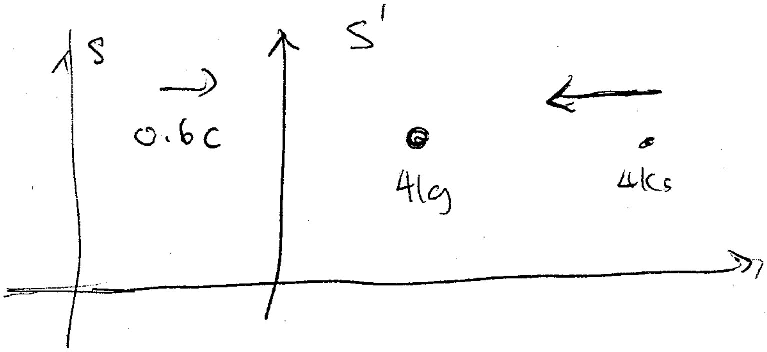
$$\Rightarrow m = \sqrt{\left(\frac{E}{c}\right)^2 - \left(\frac{p}{c}\right)^2} = \sqrt{10^2 - 0^2} = 10\ \text{kg}$$

$$> 4\text{kg} \times 2$$

Not to interpret the extra 2kg as belonging to any constituent

Let's check we get the same rest mass in another reference frame.

In S' frame



$$E' = \gamma(E - v p_x)$$

$$= \frac{1}{\sqrt{1-0.6^2}}(10c^2 - 0.6c \times 0)$$

$$= 12.5\, c^2\ \text{kg}$$

$$p_x' = \gamma\left(p_x - \frac{vE}{c^2}\right)$$

$$= 1.25\,(0 - (0.6c)(10c^2)/c^2]$$

$$= -7.5c\ \text{kg}$$

$$\Rightarrow m = \sqrt{12.5^2 - 7.5^2}\ \text{kg}$$

$$= 10\ \text{kg}$$

can try other reference frames too!